

ORDINANCE, STRUCTURE AND SYLLABUS  
of  
M.Sc. in Mathematics

(As recommended by Board of Studies (BOS) in Mathematics on 10<sup>th</sup> March 2018)  
(w. e. f. academic session 2018 -2019)

Under  
CHOICE BASED CREDIT SYSTEM  
(CBCS)



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West Bengal, INDIA

## M.A. / M. Sc. Syllabus in Mathematics

### First Year Semester- I: Mathematics

Paper Code	Title of the paper	Time allowed /week	Marks	Credit Points
MMC11	Real Analysis	4 hrs	50	4
MMC12	Complex Analysis	4 hrs	50	4
MMC13	Algebra-I	4 hrs	50	4
MMC14	Ordinary Differential Equations	4 hrs	50	4
MMC15	Partial Differential Equations	4 hrs	50	4
MMC16	Integral Transforms and Integral Equations	4 hrs	50	4
<b>Total</b>		<b>24</b>	<b>300</b>	<b>24</b>

### First Year Semester- II: Mathematics

<b>Paper Code</b>	<b>Title of the paper</b>	<b>Time allowed /week</b>	<b>Marks</b>	<b>Credit Points</b>
MMC21	Functional Analysis	4 hrs	50	4
MMC22	Topology	4 hrs	50	4
MMC23	Algebra-II	4 hrs	50	4
MMC24	Classical Mechanics	4 hrs	50	4
MMC25	Solid Mechanics and Dynamical Systems Unit I: Solid Mechanics Unit II: Dynamical Systems	4 hrs	25+25	4
MMC26	Numerical Analysis	4 hrs	50	4
<b>Total</b>		<b>24</b>	<b>300</b>	<b>24</b>

### Second Year Semester- III: Mathematics

<b>Paper Code</b>	<b>Title of the paper</b>	<b>Time allowed /week</b>	<b>Marks</b>	<b>Credit Points</b>
<b>MMC31</b>	<b>Discrete Mathematics</b>	<b>4 hrs</b>	<b>50</b>	<b>4</b>
<b>MMC32</b>	<b>Advanced Mathematical Statistics-I and Theory of Chaos</b> <b>Unit I: Advanced Mathematical Statistics-I</b> <b>Unit II: Theory of Chaos</b>	<b>4 hrs</b>	<b>25+25</b>	<b>4</b>
<b>MMC33</b>	<b>Fluid Mechanics</b>	<b>4 hrs</b>	<b>50</b>	<b>4</b>
<b>MMC34</b>	<b>Calculus of Variations and Special Functions</b> <b>Unit I: Calculus of Variations</b> <b>Unit II: Special Functions</b>	<b>4 hrs</b>	<b>25+25</b>	<b>4</b>
<b>MMC35</b>	<b>Galois Theory I and Multivariable Analysis</b> <b>Unit I: Galois Theory I (Field Extensions)</b> <b>Unit II: Multivariable Analysis</b>	<b>4 hrs</b>	<b>25+25</b>	<b>4</b>
<b>MMO31 (PX/AX)*</b>	<b>Optional Paper</b>	<b>4 hrs</b>	<b>50</b>	<b>4</b>
<b>Total</b>		<b>24</b>	<b>300</b>	<b>24</b>

## Second Year Semester- IV: Mathematics

<b>Paper Code</b>	<b>Title of the paper</b>	<b>Time allowed /week</b>	<b>Marks</b>	<b>Credit Points</b>
MMC41	Differential Geometry and Manifold Theory	4 hrs	50	4
MMC42	Operations Research	4 hrs	50	4
MMC43	Numerical Computer Laboratory	4 hrs	50	4
MMC44	Project Work	4 hrs	50	4
MME41	<p>Elective paper:</p> <p><b>Applied Stream:</b></p> <p><b>Electromagnetic Theory / Advanced Mathematical Statistics-II and Programming in MATLAB</b></p> <p><b>Unit I: Electromagnetic Theory / Advanced Mathematical Statistics-II</b> <b>Unit II: Programming in MATLAB</b></p> <p><b>Pure Stream:</b></p> <p><b>Galois Theory II and Algebraic Topology</b></p> <p><b>Unit I: Galois Theory II</b> <b>Unit II: Algebraic Topology</b></p>	4 hrs	25+25	4
MMO41 (PX/AX)	Optional Paper	4 hrs	50	4
<b>Total</b>		24	300	24

**Table-I:**  
**Optional Papers for MMO31 (PX)**

Sub-Code	Title of the Paper
P1	Advanced Complex Analysis-I
P2	Advanced Functional Analysis - I
P3	Advanced Real Analysis-I
P4	Advanced Topology-I
P5	Algebraic Coding Theory-I
P6	Algebraic Topology-I
P7	Fuzzy Mathematics-I
P8	Lie Theory of Ordinary and Partial Differential Equations
P9	Mathematical Logic-I
P10	Rings and Modules-I
P11	Theory of Computation-I

**Table-II:**  
**Optional Papers for MMO31(AX)**

Sub-Code	Title of the Paper
A1	Advanced Fluid Dynamics-I
A2	Biomathematics-I
A3	Computational Fluid Dynamics-I
A4	Differential Equations in Ecology
A5	Dynamics of Ecological System-I
A6	Dynamical Meteorology
A7	Lie Theory of Ordinary and Partial Differential Equations
A8	Magnetohydrodynamics-I
A9	Mathematical Pharmacology-I
A10	Nonlinear Differential Equations-I
A11	Quantum Mechanics-I
A12	Solid Mechanics-I
A13	Theory of Computation-I

**Table-III:**  
**Optional Papers for MMO41 (PX)**

Sub-Code	Title of the Paper
P1	Advanced Complex Analysis-II
P2	Advanced Functional Analysis - II
P3	Advanced Real Analysis-II
P4	Advanced Topology-II
P5	Algebraic Coding Theory-II
P6	Algebraic Topology-II
P7	Fuzzy Mathematics-II
P8	Mathematical Logic-II
P9	Rings and Modules-II
P10	Theory of Computation-II
P11	Weak Formulation of Elliptic Partial Differential Equation



**Table-IV:**  
**Optional Papers for MMO41(AX)**

Sub-Code	Title of the Paper
A1	Advanced Fluid Dynamics-II
A2	Biomathematics-II
A3	Computational Fluid Dynamics-II
A4	Difference Equations in Ecology
A5	Dynamics of Ecological System-II
A6	Dynamical Oceanography
A7	Magnetohydrodynamics-II
A8	Mathematical Pharmacology-II
A9	Nonlinear Differential Equations-II
A10	Quantum Mechanics-II
A11	Solid Mechanics-II
A12	Theory of Computation-II
A13	Weak Formulation of Elliptic Partial Differential Equation

**DETAILED SYLLABUS**  
**M.A./M. Sc in MATHEMATICS**

(As recommended by Board of Studies (BOS) in Mathematics on 10th March 2018)  
(To be effective from Academic Session 2018-2019)

**Semester- I: Mathematics**

**MMC11 : Real Analysis**

**Full Marks : 50 : Credit: 4**

Outer measure of a set. Properties of outer measure. Measurable sets and Lebesgue measure. Properties of measurable sets. Existence of non-measurable sets. Measurable functions and their properties. Equivalent functions. Simple functions

Sequence of functions. Almost everywhere convergence and convergence in measure. Lebesgue's theorem, Riesz theorem and Egoroff's theorem. Structure of measurable functions. Lusin's theorem and Frechet's theorem.

Necessary and sufficient condition for Riemann integrability in terms of measure. Lebesgue integral of bounded function and its properties Comparison of Riemann integral and Lebesgue integral. Passage to the limit under the sign of integration. Bounded convergence theorem.

Summable functions. Integrals of summable functions. Properties of the integrals of summable functions. Passage to the limit under the integral sign of a sequence of summable functions. Dominated convergence theorem. Levi's theorem and Fatou's theorem.

**Books Recommended:**

1. H.L.Royden, Real Analysis, Prentice Hall
2. P.R.Halmos, Measure theory, Prentice Hall
3. P.K.Jain & V.P.Gupta, Lebesgue Measure and Integration, Wiley Eastern Ltd.
4. B.Z.Vulikh, A Brief course in the Theory of Functions of a Real Variable, Mir Publishers

5. I.P.Natanson, Theory of Functions of a Real variable, Vol. 1, Frederick Ungar Pub. Co.
6. I.K.Rana, An introduction to measure and integration, Narosa Publishing House.
7. W.Rudin, Real and Complex Analysis, McGraw Hill.

### **MMC 12 : Complex Analysis**

**Full Marks : 50 : Credit: 4**

Complex integration, Cauchy's fundamental theorem (Statement only) and its consequences, Cauchy's integral formula and higher derivatives, Morera's theorem, Liouville's theorem, Fundamental theorem of algebra, Maximum modulus theorem, Schwarz's lemma, Taylor's theorem, Power Series, Radius of convergence, Cauchy-Hadamard theorem (Statement only), Zeros of an analytic functions, Uniqueness of analytic functions, Laurent's theorem, Singularities and classification of singularities, Limit points of zeros and poles, Riemann's theorem on removable singularity, Casorati-Weierstrass theorem, Cauchy's residue theorem, Evaluation of integration by residues theorem, Argument principle, Rouché's theorem, Open mapping theorem. Behavior of a function at the point at infinity, Conformal mapping, Riemann mapping theorem (Statement only), Introduction to analytic continuation, Riemann Surface.

#### **Books Recommended:**

1. R.V.Churchill and J.W.Brown, Complex variables and Applications, McGraw Hill.
2. E.T.Copson, Theory of functions of a complex variable.
3. W.Rudin, Real and Complex Analysis, McGraw Hill.
4. E.C.Titchmarsh, The theory of functions, Oxford University Press.
5. Shanti Narayan, Theory of functions of a Complex variable, S.Chand & Co., New Delhi.
6. John B. Conway, Functions of one complex variable, Springer International.
7. J.K.Lu, S.G.Zhong & S.Q.Liu, Introduction to the theory of Complex Functions, World Scientific.

**MMC13 : Algebra-I**

**Full Marks : 50 : Credit: 4**

**Linear Transformations:** Invariant subspaces of a linear transformation.

Algebra of linear transformations: The set of all linear transformations is a vector space, composition of two linear transformations, inverse of a linear transformation, singular linear transformations.

Projection: Sum, difference and product of projections.

Linear functional: Dual space, bidual space, dual basis. Natural isomorphism between and . The adjoint of a linear transformation . Annihilator of a subspace of ; = + .

Change of coordinate matrix. Effect of change of bases on the matrix representations; Any two matrix representations of are similar. A and hence any matrix representation is singular if and only if is a singular linear transformation. Matrix representation of (if it exists) and . Rank, Eigenvalues and eigenvectors of a linear transformation. Eigen vectors belonging to the distinct eigen values are linear independent. Diagonalization and eigenvectors. Characteristic polynomial of a linear transformation. is an eigen value of a linear transformation if and only if is a root of the characteristic polynomial. Minimal polynomial: Existence and uniqueness of the minimal polynomial of a linear transformation. is an eigen value of a linear transformation if and only if is a root of the minimal polynomial.

**Groups:** Direct product of groups: External direct product of groups, internal direct product of normal subgroups. Necessary and sufficient condition that a group is a internal direct product of a finite number of normal subgroups. Equivalence of internal and external direct product.

Group action: Action of a group on a nonempty set, orbits and isotropy group of. The class equation of a finite group. Burnside counting principle. Special emphasis on the action by conjugation: Conjugacy relation, the conjugacy class equation.

Cauchy's Theorem and -groups: Every finite group contains an element of order , for every prime . Converse of the Lagrange's theorem for finite commutative groups. Center of a -group is nontrivial.

Sylow Theorems: Sylow -subgroups, is a Sylow -subgroup of a finite group if and only if , where is the largest nonnegative integer such that . Every conjugate of a Sylow -subgroup is a Sylow -subgroup. Sylow theorems on the structure of finite groups. Applications of Sylow theorems to test the simplicity of some finite groups of

small order. If  $n$  and  $p$  is not prime then no group of order  $n$  is simple. Any simple group of order 60 is isomorphic to  $A_5$ .

Structure of finite abelian groups: The fundamental theorem of finite abelian groups and its application to determine the structure of some finite abelian groups of small order.

Solvable groups: Subnormal series, solvable series of subgroups. Solvable groups. Every subgroup and homomorphic image of a solvable group is solvable. If  $N$  is a normal subgroup of  $G$  such that both  $N$  and  $G/N$  are solvable, then  $G$  is solvable. A group  $G$  is solvable if and only if the  $n$ -th commutator subgroup  $G^{(n)}$ , for some positive integer  $n$ . The symmetric group  $S_n$  is solvable if and only if  $n \leq 4$ .

#### **Books recommended:**

1. Friedberg, Insel and Spence, Linear Algebra, PHI.
2. S. Kumaresan, Linear Algebra; A Geometric Approach, PHI, 2000
3. K. B. Dutta, Matrix and Linear Algebra
4. Kollman and Hill, Linear Algebra, Pearson Education.
5. P. R. Halmos, Finite dimensional Vector Spaces; D. Van Nostrand Co., Princeton.
1. Lipschutz, Linear Algebra; McGraw Hill
2. Hoffman and Kunze, Linear Algebra; PHI, New Delhi
3. P. D. Lax, Linear Algebra; John Wiley & Sons
6. Lay, Linear Algebra and its Applications, Pearson Education.
7. Durbin, Modern Algebra, John Wiley and Sons.
8. Dummit and Foote, Abstract Algebra, John Wiley and Sons.
9. D.S.Malik, J.N.Mordeson and M.K.Sen, Fundamentals of Abstract Algebra, McGraw Hill.
10. I.N.Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi.
11. Singh and Zameruddin, Modern Algebra; Vikas, New Delhi.
12. C. Musili, Introduction to Rings and Modules; Narosa.
13. Hungerford, Algebra, Springer
14. M. Artin, Algebra, PHI, 1991

**MMC 14 : Ordinary Differential Equations****Full Marks : 50 : Credit: 4**

Existence and uniqueness: Continuity theorem, strong continuity theorem, existence and uniqueness theorem for 1<sup>st</sup> order and higher order differential equations, existence and uniqueness theorems for systems of differential equations, Dependence of solutions on initial conditions and on the function.

Theory of  $n^{\text{th}}$  order homogeneous linear differential equations. Theory of  $n^{\text{th}}$  order non-homogeneous linear differential equations.

Wronskian, Abel-Liouville formula.

Adjoint and self-adjoint linear differential equations: Abel's identity, oscillatory solutions. non-oscillating solutions, estimation of distance between two successive zeros of solutions. Separation and comparison theorems.

Sturm-Liouville problems: orthogonality of characteristic functions, expansion of a function in a series of orthogonal functions.

Solutions of first and second order linear differential equations by Green's function approach for initial and boundary value problems. Green's function approach for solving integral equations.

System of linear homogeneous and non-homogeneous differential equations. Fundamental matrix. Exponential matrix function and their properties. Method of solving systems of linear ordinary differential equations by fundamental matrix and exponential matrix function.

Linear Autonomous System, Phase Plane Analysis, Equilibrium Points, Classification of equilibrium points, Stable and Unstable Points. Saddle Point, Centre, Defective Eigenvalues, Jordan Canonical Form.

Basic Introduction to Differential Inequalities, Basic Introduction to Delay Differential Equations, Basic Introduction to Fractional Differential Equations, Basic Introduction to Stochastic Differential Equations.

**Books Recommended:**

1. Differential Equations- S. L. Ross (John Wiley and Sons)
2. Elementary Differential Equations and Boundary value problems- Boyce and DiPrima (Wiley)
3. A First Course in Ordinary Differential Equations- R.E. Langer (Wiley)
4. Differential Equations with Boundary-Value Problems – Zill and Wright (CENGAGE)
5. Ordinary Differential Equations- Ince (Dover)
6. Differential Equations- H. T. H. Piaggio (G. Bell and Sons)
7. Nonlinear Ordinary Differential equations – Jordon and Smith (Cambridge)
8. J Szarski, Differential Inequalities,PWN (1965).
9. I Podlubny,Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications., Elsevier (1998).
10. Y Kuang, Delay Differential Equations: with applications in population dynamics, Academic Press, (1993).
11. B Øksendal, Stochastic Differential Equations, Springer, (2003).

**MMC 15: Partial Differential Equations**

**Full Marks: 50: Credit: 4**

Applications leading to PDEs, Well-posed PDEs, First-order PDEs, Integral surface through a given curve, Monge cone, Characteristic strip, Cauchy's method of characteristic.

Second-order PDEs with constant and variable coefficients. Monge's method of solution. Classification of second order PDE, The Canonical forms for Hyperbolic, Parabolic and Elliptic equations.

Transport equation / Advection equation - Initial value problem. Non-homogeneous equation.

Laplace's equation: Fundamental solution, Mean value formula, Properties of solutions. Solution of Laplace equation in polar coordinates.

Heat equation : Fundamental solution, Mean value formula, Properties of solutions, Energy methods.

Wave equation: Solution by spherical means, Non-homogeneous equations, Energy methods.

Representation of solutions: Separation of variables, Fourier and Laplace Transforms.

Laplace's equation in three dimensions: Solution of Laplace equation in cylindrical and spherical polar coordinates, Surface and solid harmonics.

Boundary value problems of Dirichlet and Neumann: Dirichlet's principle, Dirichlet problem for a rectangle, The Neumann problem for a rectangle, Interior Dirichlet problem for a circle, Exterior Dirichlet problem for a

circle, Interior Neumann problem for a circle, Green's function solution of Dirichlet's and Neumann's problem for sphere, formulation of Dirichlet's problems as a problem of integral equation, Poisson's integral solution.

**Books Recommended:**

1. I. N. Sneddon - Elements of Partial Differential Equations (McGraw Hill).
2. T. Amarnath - An Elementary Course in Partial Differential Equations (Narosa).
3. Lawrence C. Evans - Partial Differential Equations, Graduate Studies in Mathematics, Vol.19 ( American Mathematical Society).
4. Kenneth S. Miller - Partial Differential Equations (Krieger Pub. Co.).
5. F. John- Partial Differential Equations (Springer).
6. P. Prasad, R. Ravindran- Partial Differential Equations (John Wiley and Sons Ltd.).
7. J. David Logan - Applied Partial Differential Equations (Springer).
8. E. DiBenedetto: Partial Differential Equations (Birkhäuser).
9. A. D. Polianin, V. F. Zaitsev, Alan Moussiaux - Handbook of first Order Partial Differential Equations (CRC Press, Taylor & Francis Group).
10. Tyn Myint-U, L. Debnath - Linear Partial Differential Equations for Scientists and Engineers (Birkhäuser).

**MMC16 : Integral Transforms and Integral Equations**

**Full Marks : 50 : Credit: 4**

Integral Transforms:

Laplace transform. Convergence. Continuity requirements. Exponential order. Existence and Uniqueness. Properties of Laplace transform. Heaviside's unit step function. Convolution theorem. Complex inversion formula. Inverse Laplace transform. Properties of inverse Laplace transform.

Fourier transform. Derivation of Fourier transform from Fourier series. Properties of Fourier transform. Convolution theorem. Inverse Fourier transform. Properties of inverse Fourier transform. Parseval's Identity.

Applications of Laplace and Fourier transforms.

Integral Equations:

Basic Introduction.

Conversion of ordinary differential equations into integral equations.

Conversion of integral equations into ordinary differential equations.



Linear integral equations of Volterra type. Solution of Volterra integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition, modified Adomian decomposition and Laplace transform methods. Resolvent kernel.

Linear integral equations of Fredholm type. Solution of Fredholm integral equations by successive substitutions, successive approximations, series solution, Adomian decomposition and modified Adomian decomposition methods. Resolvent kernel. Neumann series. Solution of integral equations with separable kernels. Fredholm Alternative theorem. Hilbert-Schmidt theory of integral equations for symmetric kernels.

Singular integral equations. Solution of singular integral equations by Laplace transform method.

Existence and uniqueness of integral equations.

Applications.

#### **Books Recommended:**

1. F.G. Tricomi - Integral Equations (Inter - Science)
2. A. M. Wazwaz -Linear and Nonlinear Integral Equations. (Springer)
3. R. P. Kanwal -Linear Integral Equations (Academic Press)
4. P.P.G. Dyke- An introduction to Laplace Transforms and Fourier Series. (Springer)
5. M.G. Spiegel- Laplace Transforms (Schaum's Outlines series)
6. J. L. Schiff - The Laplace Transform (Springer)
7. S. L. Ross- Ordinary Differential Equations. (Wiley)
8. R.L. Bracewell-The Fourier Transforms and Its Applications (McGraw-Hill)

## **Semester- II: Mathematics**

### **MMC 21 : Functional Analysis**

**Full Marks : 50 : Credit: 4**

Complete metric spaces: Completion theorem, Compactness in metric spaces: Compactness, Sequential compactness countable compactness- their equivalence, Continuous function on compact metrics spaces, Lebesgue covering Lemma, Total boundedness. Stone Weierstrass theorem.

Normed linear spaces, continuity of norm function, Banach spaces; Linear operators over a normed linear space, Boundedness and continuity of linear operators, Unbounded linear operators; Norm of a bounded linear operator and its formulae. Bounded linear functionals, Dual spaces. Riesz Lemma; Compactness of closed unit ball and finite dimensionality of normed linear spaces, Hahn-Banach theorem and its applications; Open mapping theorem; Closed graph theorem; Uniform boundedness principle theorem. Weak convergence, Strong convergence.

Inner product spaces, Cauchy-Schwartz Inequality, Inner product function as a continuous function, Inner product space as a normed linear space Parallelogram law, Hilbert spaces (Separable only), Orthogonal sets in a Hilbert space, Gram-Schmidt Orthogonalization scheme, Linear independences of orthogonal system, Bessel's inequality, Parseval's equality(Statement only), Riesz representation theorem.

### **Books Recommended:**

1. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, N. Y.
2. G. Bachman & L. Narici, Functional Analysis, Academic Press
3. C. Gottman & G. Pedrick, First Course in Functional Analysis, Prentice Hall of India
4. A.H. Siddiqui, Functional Analysis with Applications, Tata-McGraw Hill
5. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill
6. B.K. Lahiri, Elements of Functional Analysis, The World Press
7. V.K. Krishnan, Text Book of Functional Analysis, Prentice Hall.
8. B.V. Limaye, Functional Analysis, Wiley Eastern.

### **MMC 22 : Topology**

**Full Marks : 50 : Credit: 4**

Cardinal numbers, Cardinal arithmetic, Order types, Well-ordered sets and ordinal numbers, Axiom of choice, Well ordering Theorem and Zorn's Lemma –their equivalence

Topological spaces; examples, union and intersection of Topologies, Discrete and Indiscrete Topologies, Weak and Strong Topologies, Base for a Topology, Necessary and sufficient condition for a base of a Topology, Sub-base, Neighbourhood system at a point, limit point of a set, closed sets and their algebra; Derived set, closure of a set, Interior, boundary of a set - their relations, Kuratowski closure operator and resulting topology, subspace and relative topology. First and second axiom spaces, Lindeloff Theorem

Continuous functions, open functions, closed functions, homeomorphism, product of finite number of spaces, sum and quotient spaces, separation axioms  $T_1$ ,  $T_2$ ,  $T_3$ , regular spaces, completely regular spaces, normal spaces, completely normal spaces, open cover, compact spaces, compact sets, F.I.P. (finite intersection property), characterization of compact sets of reals with usual topology, continuous image of compact spaces.

Connected spaces, connected sets of reals with usual topology, union of connected sets, continuous image of connected spaces, Components.

### **Books Recommended:**

1. S. T. Hu, Introduction to General Topology, Holden-Day Inc.
2. K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd.
3. Dugungi, Topology, (Prentice Hall, New Delhi)
4. G Simmons, Introduction to Topology and Modern Analysis
5. W. Thorn, Topological Structure.
6. W. J. Pervin, Introduction to General Topology (Academic Press)

### **MMC 23 : Algebra-II**

**Full Marks : 50 : Credit: 4**

**Rings:** Euclidean domain (ED) and principal ideal domain (PID) : Every Euclidean domain is a principal ideal domain.  $R$  is a field if and only if  $R$  is a Euclidean domain (principal ideal domain). Greatest common divisor (gcd) in a ring; If  $R$  is a principal ideal domain, then any two elements (not both zero) have a gcd in  $R$  and the Bezout's identity for gcd holds in  $R$ .

Prime and irreducible elements in a ring: In an integral domain every prime element is irreducible. In a PID, an element  $p$  is irreducible if and only if  $p$  is prime. If  $R$  is a PID, then an ideal  $I$  is prime (maximal) if and only if  $I = (p)$ , for some prime (irreducible) element  $p$  of  $R$ .

Unique factorization domains: Every PID is a UFD. Primitive polynomials; If  $R$  is a UFD, then the product of two primitive polynomials in  $R[x]$  is primitive. The polynomial ring  $R[x]$  over a UFD  $R$  is a UFD.

Irreducibility of polynomials over a field: Irreducible polynomials over  $F$ . Test for irreducibility of the polynomials of degree 2 or 3 on  $F$ . Eisenstein's irreducibility criterion. ( $F$  is a field)

Noetherian and Artinian rings: A ring  $R$  is Noetherian if and only if every ideal of  $R$  is finitely generated. Every homomorphic image of a Noetherian ring is Noetherian. If  $I$  be an ideal of  $R$  such that both  $R/I$  and  $I$  are Noetherian rings, then  $R$  is Noetherian. Hilbert basis theorem: If  $R$  is a Noetherian ring, then so is the polynomial ring  $R[x]$ . Every Artinian integral domain is a field.

### Canonical forms of linear transformations:

Triangular and diagonal forms: The primary decomposition theorem. A linear operator  $T$  has a triangular matrix representation if and only if the minimal polynomial of  $T$  is a product of linear factors over  $F$ . A linear operator  $T$  has a diagonal matrix representation if and only if  $T$  has a basis of eigen vectors of  $V$ . A linear operator  $T$  has a diagonal matrix representation if and only if the minimal polynomial of  $T$  is a product of distinct linear factors over  $F$ .

Jordan canonical forms: Nilpotent operators: A linear operator  $T$  is nilpotent if and only if every matrix representation of  $T$  is nilpotent. Index of nilpotency, Invariants of a nilpotent transformation; Uniqueness of the invariants. Two nilpotent linear transformations are similar if and only if they have the same invariants. Jordan blocks, Jordan canonical form. If the minimal polynomial of  $T$  is a product of linear factors over  $F$  then  $T$  is similar to a matrix in Jordan canonical form. Two linear transformations whose minimal polynomials are product of linear factors over  $F$  are similar if and only if they can be brought to the same Jordan canonical form. Rational canonical forms.

Linear transformations on inner product spaces: ( $V$  is finite dimensional inner product space) Riesz representation of the linear functional on inner product space, adjoint of a linear operator  $T$ ; matrix representation of  $T$ . Normal and self-adjoint operators. Eigen values of a self-adjoint operator are real. Unitary and orthogonal operators and their matrices. Orthogonal projections, the spectral theorem and its consequences.

Bilinear and Quadratic forms: Matrix representation of a bilinear form, Symmetric bilinear forms; A bilinear form is symmetric if and only if its matrix representation is symmetric. Diagonalization of the bilinear and symmetric bilinear forms. Orthogonal diagonalization of a real quadratic form. Positive definite negative definite and semi-definite real quadratic forms.

### Books recommended:

1. Friedberg, Insel and Spence, Linear Algebra, PHI.
11. S. Kumaresan, Linear Algebra; A Geometric Approach, PHI, 2000
12. K. B. Dutta, Matrix and Linear Algebra
13. Kollman and Hill, Linear Algebra, Pearson Education.
14. P. R. Halmos, Finite dimensional Vector Spaces; D. Van Nostrand Co., Princeton. Lipschutz, Linear Algebra; McGraw Hill
15. Hoffman and Kunze, Linear Algebra; PHI, New Delhi
16. P. D. Lax, Linear Algebra; John Wiley & Sons
17. Lay, Linear Algebra and its Applications, Pearson Education.
18. Durbin, Modern Algebra, John Wiley and Sons.
19. Dummit and Foote, Abstract Algebra, John Wiley and Sons.
20. D.S.Malik, J.N.Mordeson and M.K.Sen, Fundamentals of Abstract Algebra, McGraw Hill.
21. I.N.Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi.
22. Singh and Zameruddin, Modern Algebra; Vikas, New Delhi.
23. C. Musili, Introduction to Rings and Modules; Narosa.
24. Hungerford, Algebra, Springer
25. M. Artin, Algebra, PHI, 1991.

**MMC 24 : Classical Mechanics**

**Full Marks : 50 : Credit: 4**

Laws of Motion: Moving axes. Particle Motion Relative to the Rotating Earth. Foucault's Pendulum. Coriolis Force. Virial Theorem. Two- and Three-Body Motions.

Generalised Co-ordinates. Unilateral and Bilateral Constraints. Principle of Virtual Work. D'Alembert's Principle. Holonomic and Nonholonomic Systems. Scleronomic and Rheonomic Systems. Lagrange's Equation of Motion. Applications. Energy Equation for Conservative Fields. Cyclic or Ignorable Co-ordinates. Routh's Equations. Dynamical Systems of Liouville's Type Hamilton's Equations of Motion. Calculus of Variations. Hamilton's Principle. Lagrange's

Equations of Motion from Hamilton's Principle. Principle of Least Action. Constants of Motion. Noether's Theorem. Conservation Laws. Infinitesimal transformations.

Motion of a Rigid Body about a Fixed Point in it. Euler's Dynamical Equations. Eulerian angles. Gyroscope and nonholonomic Problems. Motion of a Symmetrical Spinning Top on a perfectly Rough Floor. Stability of Steady Precession.

Canonical Transformations. Generating Functions. Poisson's Bracket. Jacobi's Identity. Poisson's Theorem. Jacobi-Poisson Theorem.

Hamilton-Jacobi Equation. Jacobi's Theorem. Hamilton's Principal Function. Hamilton's Characteristic Function. Action-Angle Variables. Adiabatic Invariance.

Theory of Small Oscillations (Conservative System). Normal Co-ordinates. Oscillations under Constraints. Stationary Character of Normal Modes. Elements of Non-linear Oscillations.

An introduction to Quantum Mechanics.

#### **Books Recommended:**

1. H.Goldstein, Classical Mechanics. Narosa Publishing House, New Delhi, (1980).
2. F.Gantmacher, Lectures in Analytical Mechanics, MIR Publishers, Moscow 1975)
3. J.L.Synge and B.A. Griffith, Principles of Mechanics, McGraw-Hill, N.Y. (1970)
4. N.C.Rana and P.S.Joag, Classical Mechanics, Tata McGraw Hill Pub. Company Ltd., New Delhi (1998)
5. N.H.Louis and Janet D.Finch, Analytical Mechanics, C.U.P. (1998)
6. E.T.Whittaker, A Treatise of Analytical Dynamics of Particle and Rigid Bodies, C.U.P. (1977)
7. S.W.McCusky, An Introduction to Advanced Dynamics, Addison-Wesley Publ. Co. Inc. Massachusetts (1953)
8. A.S.Ramsey, Dynamics Part-II, C.U.P. (1972)
9. L.Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers. Moscow (1973)
10. R.H.Dicke and J.P.Wittke, Introduction to Quantum Mechanics, Addison Wesley, (1960)
11. Rydnik, ABC of Quantum Mechanics, Peace Publisher, Moscow.
12. F.Chorlton, Textbook of Fluid Dynamics, CBS Publications, Delhi, 1985.
13. A.S. Ramsey - Newtonian Attractions. (Cambridge)
1. O.D. Kellog – Foundations of Potential Theory, Dover (1963).

**MMC 25 : Solid Mechanics and Dynamical Systems**

**Full Marks : 50 : Credit: 4**

**Unit I : Solid Mechanics**

**Full Marks : 25 : Credit: 2**

Notion of a Continuum and of Deformable Bodies. Linear Elastic Solid (or Hookean Solid).

Analysis of Stress: Body and surface forces. Vector stress (or stress vector) and Notation for its Components. Specification of stress at a point. Stress Tensor. Equations of Equilibrium. Symmetry of Stress Tensor. Surface Boundary Conditions. Rule of Transformation of Stress Components. Principal Stresses and Stress Invariants. Stress quadric of Cauchy. Maximum normal and Shearing Stresses. Mohr's Diagram. Problems

Analysis of Strain: Strain Tensor. Finite Strain Components. Infinitesimal Strain and Rotation Components. Geometrical Interpretation of Infinitesimal Strain Components. Transformation of Infinitesimal Strain Components. The Strain Quadric. Principal Strains and Principal Axis of Strain. Rate of Deformation Tensor. Problems.

Motion of Deformable Bodies: Lagrangian and Eulerian Descriptions. Path line and Stream line. Material Derivative, Condition on a Boundary Surface. Conservation of Mass. The Continuity Equation. Momentum Principles. Equation of Motion. Energy Balance. Laws of Thermodynamics, Equation of State. Generalised Hooke's law. Isotropy. Elastic Moduli.

### **Books Recommended:**

1. I. S. Sokolnikoff, *Mathematical Theory of Elasticity* (McGraw Hill, 1956)
2. T. J. Chung, *Continuum Mechanics* (Prentice Hall, 1988)
3. S. C. Hunter, *Mechanics of Continuous Media* (Ellis Horwood Ltd., England, 1983)
4. A. J. M. Spencer, *Continuum Mechanics* (Longman, 1980)

## Unit II : Dynamical Systems

Full Marks : 25 : Credit: 2

### Continuous Dynamical Systems:

**Nonlinear Systems:** Linearization of Non-linear Systems, Limitations, Hartman–Grobman Theorem, Local Stability, Global Stability, Lyapunov Function, Lyapunov Theorem on Stability, LaSalle Invariance Principle,

**Oscillations:** Limit Set, Attractors, Periodic Orbits, Limit Cycle, Poincare-Bendixson Theorem, Bendixson-Dulac Criterion.

### Discrete Dynamical Systems:

Maps and Flows, Composition of Maps, Orbits, Phase Portrait, Fixed Points, Stable and Unstable Fixed Points, Basin of Attraction and Basin Boundary, Linear Stability Analysis, Cobweb Diagram, Periodic Points, Periodic Cycles, Stability of Periodic Point and Periodic Cycle, Hyperbolic Points, Non-Hyperbolic Points, Schwarzian Derivative.

### Some Maps:

Tent Map, Logistic Map, Dynamics of Quadratic Maps, Shift Map, Baker's Map, Circle Map, Henon Map, Smale Horseshoe Map.

**Conjugacy of Maps:** Conjugacy, Topological Semi-conjugacy, Homeomorphism, Topological Conjugacy, Conjugacy Between different Maps.

**Center Manifold and Normal Form Theory:** Center Manifolds, Properties of Center Manifolds, Center Manifolds Depending on Parameters, Normal Forms.

### Books Recommended:

1. S Strogatz - Nonlinear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry, And Engineering, Hachette (2014).
2. G C Layek - An Introduction to Dynamical Systems, Springer (2015).
3. P Glendinning - Stability, Instability and Chaos, Cambridge (1994).
4. S Wiggins - Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer (1996).
5. M W Hirsch, S Smale, R L Devaney - Differential Equations, Dynamical Systems, and an Introduction to Chaos, Academic Press (2012).
6. R A Holmgren - A First Course in Discrete Dynamical Systems, Springer (1996).
7. J Hale and H Koack - Dynamics and Bifurcations, Springer (2012).



Interpolation: Polynomial Interpolation: Divided Differences and their properties, Generalized Newton's divided difference interpolation formula, Piecewise polynomial interpolation, Hermite interpolation formula, Cubic spline interpolation and its convergence.

Approximation of Functions: Norms, Least-squares polynomial approximation, Orthogonal polynomials, Gram-Schmidt orthogonalization, Chebyshev polynomials, Uniform approximation in mini-max sense of error, Lanczos economization.

Numerical Integration: Errors in Newton-Cotes quadrature formula, Gauss-Legendre and Gauss-Chebyshev quadrature formulae, Romberg integration.

Matrix Eigenvalues and Eigenvectors: LU decomposition of square matrices, Power method with shifting, Deflation method; Jacobi's method, Givens' method and Householder's for symmetric matrices.

Numerical Solution of Initial Value problems (IVP) for ODE: First Order Equations: Single-step Methods: Taylor's series, Euler, Modified Euler, 4<sup>th</sup> order Runge-Kutta (R-K) method; Convergence of single-step method. Multi-step Predictor-Corrector Methods: Adams-Bashforth method, Adams-Moulton method, Milne's method --- Convergence and stability.

Two-Point Boundary Value problems for ODE: Shooting method, Finite difference method.

Numerical Solution of Partial Differential Equations (PDE) by Finite Difference Methods: Classification of second order PDE, Representation of derivatives by forward, backward and central differences, Thomas Algorithm, Parabolic Equation (Heat-Conduction Equation) (1D) by explicit finite difference and implicit Crank-Nicolson methods, Poisson Equation on a rectangular region, Hyperbolic Equation (Wave Equation) in one space dimension.

Books Recommended:

1. L. W. Johnson and R. D. Riess, Numerical Analysis, Addison-Wesley Publishing Company, Inc., USA (1977).
1. A. Ralston and P. Rabinowitz, A First Course in Numerical Analysis, McGraw-Hill, N. Y (1978).
1. K. E. Atkinson, An Introduction to Numerical Analysis, John Wiley and Sons (1989).
2. A. Gupta and S. C. Bose, Introduction to Numerical Analysis, Academic Press (1989).
3. P. Niyogi, Numerical Analysis and Algorithms, Tata McGraw-Hill, New Delhi (2004).
1. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publisher, New Delhi (2007).

### **Semester- III: Mathematics**

Symbolic Logic: Propositional Logic --- Propositional language, truth (by tables), tautologies, contradictions, validity, adequacy of connectives. Axiom system, Deduction theorem, derivations, soundness and completeness theorems. Independence of axioms, consistency.

Predicate Logic --- First order language, translations from natural language to first order language and vice-versa. Formal semantics, satisfaction, truth, validity, models. A set of axioms and models. Soundness theorem. Completeness theorem (Statement only).

Graphs: Definitions and basic concepts, problems in which Graphs provide a mathematical model - Konigsberg Bridge problem, Connector problem, Utilities problem, Travelling salesman problem, Shortest path problem. Vertex degrees and incidence---first theorem of graph theory. Sub-graphs, Graph isomorphism, Paths and cycles. Connectedness and components. Matrix representation of graph.

Euler tour, Euler graph – the Konigsberg Bridge problem. Hamiltonian path, Hamiltonian cycle and Hamiltonian graph- the traveling salesman problem.

Trees --- definition and basic concepts. Spanning trees. Connector problem --- Kruskal's algorithm. Shortest path problem --- Breadth First Search algorithm, Depth First Search algorithm. Dijkstra's algorithm.

Cut-sets and cut-vertices — connectivity. (2) Planar graphs—Euler's formula. Dual of a planar graph.(4) Vertex colouring, edge colouring and map colouring --- the four colour conjecture .

Combinatorics: The basic problem of combinatorics — counting, Enumeration of permutations and combinations. Principle of inclusion and exclusion – examples only. The pigeon-hole principle – examples only.

Recursions. Generating functions and recurrence relations. Solution of recurrence relation by generating function technique — problem solving.

Partially ordered sets, lattices, chains and anti-chains — Dilworth's theorem. The necklace problem —Burnside's theorem. Polya's theorem and their applications.

### Books Recommended:

1. M. Huth, M. Ryan --- Logic in Computer Science, 2<sup>nd</sup> edition, Cambridge University Press, 2004.
2. J. L. Mott, A. Kandel, T. P. Baker—Discrete mathematics for Computer Scientists and Mathematicians (2<sup>nd</sup> edition), Prentice-Hall (India), 1999.
3. B. Kolman, R. C. Busby, S. Ross—Discrete mathematical structures, (3<sup>rd</sup> edition), Prentice Hall, 1999.
4. E. Mendelson --- Introduction to Mathematical Logic, 5<sup>th</sup> edition, N.Y., Van-Norstand, 1997.
5. I. M. Copi, C. Cohen --- Introduction to logic, 9<sup>th</sup> edition, Prentice Hall (India), 1997.
6. J. P. Tremblay, R. Manohar – Discrete Mathematical Structures with Applications to Computer Science, McGraw Hill, 1997.
7. J. Kelley --- The essence of logic, Prentice Hall (India), 1997.
8. J. H. Van Lint, R. M. Wilson — A course in Combinatorics, Cambridge University Press, 1992.
9. A. Margaris --- First order mathematical logic, Dover Publications, N.Y., 1990.
10. M. O. Albertson, J. P. Hutchinson – Discrete mathematics with algorithms, John Wiley and Sons, 1988.
11. C. L. Liu—Elements of Discrete Mathematics, 2<sup>nd</sup> edition, McGraw Hill, Computer Science series, 1986.
12. N. Deo — Graph theory with applications to Engineering and Computer Sciences, Prentice Hall (India).
13. R. A. Brualdi—Introductory Combinatorics, Prentice Hall (India).

**MMC32 : Advanced Mathematical Statistics-I and Theory of Chaos    Full Marks : 50 : Credit: 4**

**UNIT I:Advanced Mathematical Statistics – I**

**Full Marks : 25    : Credit: 2**

**Non-parametric tests:** One sample test --- Wilcoxon signed-rank test; Two sample test --- Wilcoxon-Mann-Whitney two-sample rank sum test. Confidence intervals for medians.

**Gauss-Markov model:** Gauss-Markov model --- assumptions. Gauss-Markov theorem --- estimability of parameters, BLUE.

**Regression:** The linear model. Covariance, correlation and their significances.

**Analysis of variance:** Different ANOVA models --- Fixed, random and mixed effects models. One-way and two-way classifications --- Classical, Bayesian and Interval estimations. Simple examples.

**Books Recommended:**

- 1.Aitken, A. C. (1944). Statistical Mathematics. 3rd edn. Edinburgh and London: Oliver and Boyd.
- 2.Feller, W. (1968). An Introduction to Probability Theory and Its Applications, Volume I. New York: Wiley.
- 3.Feller, W. (1971). An Introduction to Probability Theory and Its Applications, Volume II. New York: Wiley.
- 4.Fisher, R. A. (1922), On the mathematical foundations of theoretical statistics. Reprinted in Contributions to Mathematical Statistics (by R. A. Fisher) (1950), J. Wiley & Sons, New York.
- 5.Hogg, R. V. and Craig, A. T. (1978). Introduction to Mathematical Statistics. New York: Macmillan.
- 6.Taylor, L. D. (1974). Probability and Mathematical Statistics. New York: Harper & Row.
- 7.Sahoo, P. (2013). Probability and mathematical Statistics. Louisville, USA.

## UNIT II: Theory of Chaos

Full Marks : 25 : Credit: 2

### Theory of Bifurcations:

Bifurcations in One-Dimensional Systems: Saddle-Node Bifurcation, Pitchfork Bifurcation, Transcritical Bifurcation.

Bifurcations in Two-Dimensional Systems: Saddle-Node Bifurcation, Pitchfork Bifurcation, Transcritical Bifurcation, Hopf-Bifurcation, Homoclinic and Heteroclinic Bifurcations.

Period Doubling Bifurcation, Neimark-Sacker Bifurcation.

**Chaos:** Sensitive Dependence on Initial Conditions (SDIC), Sarkovskii's Theorem, Period-Three Implies Chaos for 1-D Maps. Some Chaotic Maps, Universal Sequence, Feigenbaum Number, Poincaré Section, Lyapunov Exponents, Routes of Chaos, Some Examples of Chaos.

**Fractals:** Self-similar Fractals, Constructions of Self-similar Fractals, Dimensions of Fractals, Strange Attractor, Strange Repeller.

### Books Recommended:

1. S Strogatz - Nonlinear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry, And Engineering, Hachette (2014).
26. G C Layek - An Introduction to Dynamical Systems, Springer (2015).
27. P Glendinning - Stability, Instability and Chaos, Cambridge (1994).
28. S Wiggins - Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer (1996).
29. M W Hirsch, S Smale, R L Devaney - Differential Equations, Dynamical Systems, and an Introduction to Chaos, Academic Press (2012).
30. R A Holmgren - A First Course in Discrete Dynamical Systems, Springer (1996).
31. J Hale and H Koack - Dynamics and Bifurcations, Springer (2012).
- 32.

**MMC 33 : Fluid Mechanics****Full Marks : 50 : Credit: 4**

Fluids: Ideal Frictionless Fluid. Linearly Viscous Fluid. Stokes' Condition. Different types of flows (Laminar and Turbulent, Steady and Unsteady, Uniform and Non-uniform, Rotational and Irrotational) .

Kinematics: Lagrangian and Eulerian descriptions. Velocity and acceleration of a fluid particle. Equation of continuity or conservation of mass. Boundary surfaces. Stream lines. Path lines and streak lines. Velocity potential. Vortex lines. Irrotational and rotational motions. Simple problems.

Equations of Motion of Inviscid Incompressible Fluid: Lagrange's and Euler's equations of motion. Integrals of Equation of Motion, Kelvin's circulation theorem. Helmholtz's vorticity equation. Principle of permanence of Irrotational Motion. Equations referred to moving axes. Impulsive actions. Stream function. Simple problems.

Irrotational motion in two dimensions. Complex velocity potentials. Sources, sinks, doublets and their images. Conformal mapping. Milne-Thomson circle theorem. 2D irrotational motion produced by motion of circular, co-axial and elliptic cylinders in an infinite mass of liquid. Kinetic energy of liquid. Theorem of Blasius.

General theory of stress and rate of strain: Newton's law of viscosity. Newtonian and non-Newtonian fluids. Stress vector and components of stress tensor. Plane stress. Normal and shearing strain. Constitutive equations for a compressible Newtonian fluid. Navier-Stokes equations of motion of a viscous incompressible fluid. Some exact solutions. Steady motion between parallel plates. Dissipation of energy. Simple problems.

**Books Recommended:**

1. A.S.Ramsey, A Treatise of Hydrodynamics. Part-II, G. Bell & Sons Ltd., 1957
2. Milne Thomson, Theoretical Hydrodynamics, Macmillan, 1960.
3. G.K. Batchelor- Introduction to Fluid Dynamics, Cambridge University Press, 1967
4. L.D. Landau and E. M. Lifschitz- Fluid Mechanics, Eng. Trans. Pergamon Press, 2<sup>nd</sup> edition" 1987
5. F. Chorlton - Text Book of Fluid Dynamics, Van Norstrand, 1967.

**MMC 34 : Calculus of Variations and Special Functions**

**Full Marks : 50 : Credit: 4**

**Unit I: Calculus of Variations**

**Full Marks : 25 : Credit: 2**

Variational problems with fixed boundaries: The concept of variation and its properties. Euler's equation. Functionals dependent on higher order derivatives. Functionals dependent on functions of several independent variables. Applications.

Variational problems with moving boundaries: Transversality conditions. Orthogonality conditions. Variational problem with a moving boundary for a functional dependent on two functions. One sided variations. Applications.

Sufficient conditions for extremum: Field of extremals. Weak and strong extremum. Jacobi condition. Weirstrass function. Legendre condition. Applications.

Variational problems with subsidiary conditions: Isoperimetric problems. Theory of optimal control. Applications.

Direct method in variational problems: Euler method of finite difference. Rayleigh-Ritz method. Galerkin method. Kantorovich method.

**Books Recommended:**

1. I. M. Gelfand and S. V. Fomin - Calculus of Variations (Prentice Hall)
9. A. S. Gupta - Calculus of Variations with Applications (Prentice Hall)
10. R. Weinstock- Calculus of Variations (Dover Publications)

## Unit II: Special Functions

Full Marks : 25 : Credit: 2

Solution of differential equations in complex plane. Series solution of Legendre's equation, Bessel's equation, Hypergeometric equation, Laguerre's equation and Hermite's equation.

Legendre functions, Legendre polynomials and their zeros, Rodrigues' formula, generating function, Legendre coefficients, Recurrence relations, Orthogonality.

Bessel functions, Bessel function of first kind and of second kind of integer order, Generating function, Identities, Recurrence relations, Orthogonality, Bessel integral formula.

Laguerre polynomials, Generating function, Recurrence relations , Orthogonality.

Hermite polynomials, Generating function, Recurrence relations , Orthogonality.

Hypergeometric function and its properties.

Dirac-delta function and its properties.

### Books Recommended:

1. Special Functions for Scientists and Engineers, W.W. Bell (Dover).
2. Special Functions & Their Applications, N. N. Lebedev, R. A. Silverman (Dover)
3. Introduction to Bessel Functions, F Bowman(Dover)
4. An Introduction to Special Functions, C. Viola (Springer)
5. Special Functions - Z. X. Wang, D. R. Guo, (World Scientific)

**MMC 35 : Galois Theory I and Multivariable Analysis**

**Full Marks : 50 Credit: 4**

**Unit I: Galois Theory I (Field Extensions)**

**Full Marks : 25 : Credit: 2**

**Field extension:** Finite field extension, Algebraic and transcendental extension, Algebraically closed fields.

**Splitting fields:** Splitting fields, Existence and uniqueness (upto isomorphism),

**Separable extensions:** Separable and inseparable extensions. Perfect fields, Simple extension, Primitive elements, Galois fields.

**Books Recommended:**

1. D. S. Malik, J. N. Mordeson and M. K. Sen, Fundamentals of Abstract Algebra, McGraw- Hill. 1997.
2. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. 1975.
3. I. Stewart, Galois Theory, Chapman and Hall, 1989.
4. J. P. Escofier, Galois Theory, GTM-204, Springer, 2001.

**Unit II: Multivariable Analysis**

**Full Marks : 25 : Credit: 2**

Introduction to the function where . Limits and Continuity, Matrix of partial derivatives, Total derivatives, Consequences of Differentiability, Conditions for Differentiability, and its properties. Implicit Function Theorem, Inverse Function Theorem, Iterated Partial Derivatives, Taylor's Theorem, Stationary Points, Constrained Maxima and Minima, Differentiation in vector spaces, Introduction of Gateaux and Frechet Derivative

**Books Recommended:**

1. P. Mikusinski, M.D. Taylor ---An introduction to Multivariable Analysis, Birkhauser 2002.
2. Sean Dineen - Multivariate calculus and geometry , Third Ed. Springer 2014.



**MMO-31(PX/AX)**

**Full Marks : 50      Credit: 4**

**Optional Paper**

## SEMESTER-IV

### MMC41 : Differential Geometry and Manifold Theory

Full Marks : 50 : Credit: 4

Curvilinear coordinates in  $E^n$ . Base vectors and reciprocal base vectors. Interpretation of covariant derivatives. Intrinsic derivative, Parallel vector fields. Serret-Frenet formulas.

First fundamental quadratic form. Angle between two intersecting curves in a surface. Isometric surfaces, Gaussian curvature, Geodesic. Geodesic coordinates. Bianchi's identity, Ricci tensors, Einstein space. Geodesic curvature of surface curves.

The second fundamental form of a surface. Tensor derivative. Formulas of Gauss, The integrability conditions. Formulas of Weingarten, Equations of Gauss and Codazzi. Mean and total curvature of a surface. Meusnier's theorem. Principal directions and principal curvatures.

#### Books Recommended:

1. T.J. Willmore, An introduction to Differential and Riemannian Geometry, Oxford University Press
2. J.A.Thorpe, Introduction to Differential Geometry, Springer-Verlag.
3. B.O.'Neil, Elementary Differential Geometry, Academic Press
4. S.Sternberg, Lectures on Differential Geometry, Prentice-Hall

**MMC42 : Operations Research****Full Marks : 50 : Credit: 4**

Operations Research --- an overview. Revised Simplex Method --- minimization and maximization problem. Sensitivity Analysis --- Change in profit (or cost) contribution coefficients, change in availability of resources. change in input-output coefficients.

Integer Linear Programming --- Branch and Bound algorithm, Cutting plane algorithm.

Non-linear Programming --- Formulation of Non-linear programming problem --- Graphical method of solution. Unconstrained optimization. Optimization with equality constraints. Kuhn-Tucker conditions for constrained optimization. Convex programming. Quadratic programming Problems by (i) Wolfe's method and (ii) Beale's method.

Dynamic Programming --- Deterministic and probabilistic models.

Inventory --- Introduction, Features of inventory system, Inventory model building. Deterministic models with (i) No Shortage, (ii) Shortage. Multi item inventory models with constraints. Probabilistic models --- Single period probabilistic models (i) without set up cost, (ii) with set up cost.

Queuing Theory --- Introduction. Essential features of Queuing system. Probability distribution in Queuing Models. Classification of Queue models. Solution of Queuing models: [1]  $\{(M/M/1):(infinity/FCFS)\}$ , [2]  $\{(M/M/1):(n/FCFS)\}$  [3]  $\{(M/M/s):(infinity/FCFS)\}$  [4]  $\{(M/M/s):(n/FCFS)\}$ .

**Books Recommended:**

1. H. A. Taha-Operations Research-An Introduction. Macmillan Pub. Co., Inc., New York.
2. G. Hadley -Nonlinear and Dynamic Programming, Addition-Wesley.
3. S. S. Rao - Optimization Theory and Application, Wiley Eastern.
4. K Sarup, P. K. Gupta and Man Mohan - Operation Research, Sultan Chand & Sons.
5. J. K. Sharma-Operation Research, Mcmillan India.
6. S. D. Sharma-Operation Research, Kedarnath & Ramnath, Meerat.
7. O. L. Mangasarian-Non linear Programming, McGraw Hill.
8. Peressini, Sullivan and Uhl -The mathematics of Nonlinear programming, Springer-Verlag.
9. Rabindran, Phillips, Solberg -Operation Research, John Wiley & Sons.

**MMC 43 : Numerical Computer Laboratory****Full Marks : 50 : Credit: 4**

Programming in C: Introduction to C. Data types, variables and constants. Operators and Expression: Precedence and Associativity. Unary plus and Minus operators. Binary Arithmetic operators, Arithmetic assignment operators, Relational operators, Logical operators, increment and decrement operators, conditional operators. Arithmetic expressions, Cast operator. Statements --- Assignment statement, Input/output statements. Control Statements: Decision making with if statement, if else statement, Nesting of if else statements, switch statement, goto statement, while statement, do statement, for statement, break and continue statements.

Note: It should be learnt by practice only. No theoretical questions should be asked. Programming should preferably be taught in the Lab. Only a few important points and structures may be discussed in the class, and implemented on the computer. Students should learn the following numerical methods and implement in C on a computer.

- (1) Interpolation by --- Newton's divided difference formula, Cubic Spline
- (2) Integration by --- Romberg's formula, Gaussian quadrature formula, Chebyshev's quadrature formula
- (3) Eigen values and Eigen vectors of a matrix by --- Power method, Jacobi's method
- (4) Matrix inversion by LU- decomposition
- (5) Finding solution of Initial Value Problem ( IVP ) for ODE by --- Runge-Kutta method, Adams method, Milne's method
- (6). Finding solution of two-point Boundary Value Problems ( BVP ) by Shooting method
- (7) Finding solution of Partial Differential Equations ( PDE ) by Finite Difference method
- (8) Numerical evaluation of Integral Equation

**Books Recommended:**

Balaguruswamy- Numerical Methods

1. E. Balaguruswamy-Programming in ANSI C., Tata -McGraw Hill (1992)
2. B.S. Gottfried- Theory and Problems of Programming in C, Tata McGraw Hill (1998).
3. V. Rajaraman, Programming in C, Prentice Hall (1994)
4. E. Balaguruswamy-Numerical Methods, Tata -McGraw Hill.
5. S. Arumugam, A.T. Isaac, A. Somasundaram- Numerical Methods, SCITECH.

**MMC44: Project Work**

**Full Marks : 50 : Credit: 4**

**MME 41 (Applied Stream) :**

**Electromagnetic Theory / Advanced Mathematical Statistics-II  
and Programming in MATLAB Full Marks : 50 : Credit: 4**

**UNIT-I: Electromagnetic Theory Full Marks : 25 : Credit: 2**

**Electrostatics:** Coulomb's law, Forces and fields, Electric field and potential, Fields due to continuous and discrete charges, Gauss' law and applications, Poisson and Laplace equations, Electric dipole, Dielectric media, Polarization, Electrostatic energy, Simple electrostatic boundary value problems.

**Magnetostatics:** Steady current, Lorentz force, Magnetic field intensity, Equation of continuity, Biot-Savart's law, Magnetic vector potential, Ampere's circuit law, Magnetization, Magnetic energy density

**Electrodynamics:** Faraday's law, Displacement current, Maxwell's equations for electromagnetic field and their empirical basis, Electromagnetic potential, Electromagnetic energy, Poynting theorem. Plane electromagnetic waves in dielectric and conducting medium, Field of a point charge in uniform motion.

**Books Recommended:**

1. Introduction to Electrodynamics- David J. Griffiths (Prentice Hall)
2. Classical Electrodynamics- J. D. Jackson (John Wiley and Sons)
3. Electricity and magnetism- Chattopadhyay and Rakshit (New Central)
4. Basic laws of Electromagnetism – I. E. Irodov (CBS)
5. Electromagnetism – B. B. Laud (New Age International)
6. Foundations of Electromagnetic theory-J. R. Reitz, F. J. Milford and R. W. Christy, Addison Wesley, 1966.

**UNIT-I: Advanced Mathematical Statistics-II Full Marks : 25 : Credit: 2**

**Multivariate distribution:** Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters --- partial and multiple correlation coefficients and related tests. Data reduction techniques --- Principle component analysis, Discriminant analysis, Cluster analysis, Canonical correlation. Simple social and physical problems.

**Randomized design:** Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD.  $2^k$  factorial experiments --- confounding and construction.

**Books Recommended:**

1. Aitken, A. C. (1944). *Statistical Mathematics*. 3rd edn. Edinburgh and London: Oliver and Boyd.
2. Feller, W. (1968). *An Introduction to Probability Theory and Its Applications, Volume I*. New York: Wiley.
3. Feller, W. (1971). *An Introduction to Probability Theory and Its Applications, Volume II*. New York: Wiley.
4. Fisher, R. A. (1922), On the mathematical foundations of theoretical statistics. Reprinted in *Contributions to Mathematical Statistics* (by R. A. Fisher) (1950), J. Wiley & Sons, New York.
5. Hogg, R. V. and Craig, A. T. (1978). *Introduction to Mathematical Statistics*. New York: Macmillan.
6. Taylor, L. D. (1974). *Probability and Mathematical Statistics*. New York: Harper & Row.
7. Sahoo, P. (2013). *Probability and mathematical Statistics*. Louisville, USA.

## **MME 41 (Applied Stream) :**

### **Unit II: Programming in MATLAB Full Marks : 25 : Credit: 2**

#### **1. Introduction to MATLAB:**

- a. Matlab basics
  - i. The MATLAB environment, Basic computer programming, Variables and constants
  - ii. Operators and simple calculations, Formulas and functions
  - iii. MATLAB toolboxes
- b. Matrices and vectors
  - i. Vectors and matrices in MATLAB
  - iv. Matrix operations and functions in MATLAB
  - v. Exercises

#### **2. MATLAB Programming**

- i. Algorithms and structures
- ii. MATLAB scripts and functions (m-files)
- iii. Simple sequential algorithms
- iv. Control structures (if...then, loops)
- v. Reading and writing data, file handling
- vi. Personalized functions
- vii. Toolbox structure
- viii. MATLAB graphic functions
- ix. Exercises

#### **3. Numerical Simulations**



- i. Simulation of systems of ordinary differential equations
- x. Simulation of partial differential equations: Wave equations
- xi. Solution of nonlinear equations: Boundary value problems--Poisson and Laplace equations, solutions of two coupled (algebraic, trigonometric) nonlinear equations, Roots of a polynomial etc.
- xii. Exercises

**Books Recommended:**

1. A guide to Matlab for beginners and experienced users, B. R. Hunt, R. L. Lipsman and J. M. Rosenberg, Cambridge University Press, 1995.
2. Applied numerical methods using Matlab, W. Y. Yang, W. Cao, T-S Chun and J. Morris, Wiley-Interscience, John Wiley & sons, 2005.
3. Basics of Matlab and beyond, A. Knight, Chapman & Hall/CRC, 2000.
4. Fundamentals of Electromagnetics with MATLAB, K. E. Lonngren, S. V. Savov, R. J. Jost, SciTech Publishing, 2007.

**MME 41(Pure Stream) : Galois Theory II and Algebraic Topology Full Marks: 50 Credit: 4**

**Unit I: Galois Theory II**

**Full Marks : 25 : Credit: 2**

**Galois Theory:** Normal extensions Automorphisms of extensions, Galois extensions, Fundamental theorem of Galois theory, Galois group of a polynomial.

**Solvability by radicals:** Solution of polynomial equations by radicals, Insolvability of the general equation of degree 5 by radicals. Cyclotomic polynomials and the Wedderburn Theorem on division ring.

**Geometric constructions:** Geometric constructions by ruler and compass; Three famous impossibilities. A regular  $n$ -gon is constructible if and only if  $n$  is a Fermat prime.

**Books Recommended:**

1. D. S. Malik, J. N. Mordeson and M. K. Sen, Fundamentals of Abstract Algebra, McGraw Hill. 1997.
2. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. 1975.
3. I. Stewart, Galois Theory, Chapman and Hall, 1989.
4. J. P. Escofier, Galois Theory, GTM-204, Springer, 2001.

**Unit II: Algebraic Topology**

**Full Marks : 25 : Credit: 2**

Homotopy of paths; contractible spaces and homotopy type; Fundamental group and its properties; Simply connected space; Fundamental group of circle; Simplicial Complexes; Polyhedra and triangulations; Simplicial Approximation

Books:

1. Algebraic topology – Satya Deo.
2. Topology – James R Munkres.

### Optional Papers (Pure Stream)

MMO 31 (P1): Advanced Complex Analysis-I

Full Marks : 50 : Credit: 4

Harmonic functions, Gauss' mean value theorem for harmonic functions, Characterization of harmonic functions by mean value property, Poisson's integral formula, Harnack's inequality, Maximum and minimum principles, Dirichlet problem for a disc, Existence and uniqueness of the solution of Dirichlet problem for a disc.

Maximum Modulus theorem (Statement only), Properties of  $M(r)$  and  $A(r)$ , Connection of the degree of a polynomial in relation to  $M(r)$  and  $A(r)$ , Hadamard's three circles theorem, Theorem of Borel and Caratheodory.

Entire functions, Weierstrass factorization theorem, Order and type of an entire function, The function  $n(r)$ , Jensen's formula, Zeros of entire functions, Exponent of convergence, Canonical product, Genus, Borel's first theorem, Borel's second theorem (Statement only), Picard's theorem, Hadamard's factorization theorem, Order and type of an entire function in terms of its Taylor's coefficients, Idea of fixpoints.

#### Books Recommended:

1. H. Cartan, Elementary theory of analytic functions of one or several complex variables, Dover Publications, INC, New York
9. E. C. Titchmarsh, The theory of functions, 2nd ed. Oxford University press.
10. G. Valiron, Lectures on the general theory of integral functions, Chelsea Pub. Co. .
11. B. Ja Levin, Distribution of zeros of entire functions, American Mathematical Society Providence.
12. L.V. Ahlfors, Complex Analysis
13. R. P. Boas, Entire Functions
14. A.I. Markusevich, Theory of Functions of a Complex Variables, Vol. I & II

**MMO 41 (P1): Advanced Complex Analysis-II**

**Full Marks : 50 : Credit: 4**

Meromorphic functions, Mittag-Leffler's theorem Poisson-Jensen formula, Definitions of functions  $m(r, a)$ ,  $N(r, a)$  and  $T(r, f)$ , Nevanlinna's first fundamental theorem, Cartan's identity, Convexity theorem, Orders of growth, Order of a meromorphic function, Comparative growth of  $T(r, f)$  and  $\log M(r, f)$ . Nevanlinna's second fundamental theorem, Estimation of  $S(r, f)$  (Statement only), Deficient values, Upper bound of the sum of deficiencies, Deficient functions, Nevanlinna's theorem on deficient functions(Statement only), Idea of value sharing. Nevanlinna's five-point uniqueness theorem, Milloux's theorem, Milloux's basic results.

Elliptic functions, Weierstrass elliptic function  $\wp(z)$ , Addition theorem for  $\wp(z)$ , Differential equation satisfied by  $\wp(z)$ , the numbers  $e_1, e_2, e_3$ .

**Books Recommended:**

1. W. K. Hayman- Meromorphic Functions, Oxford University Press.
2. C. C. Yang and H. X. Yi, Uniqueness theory of meromorphic functions, Kluwer Academic Press.

**MMO 31 (P2) : Advanced Functional Analysis-I**

**Full Marks : 50 : Credit: 4**

Topological Vector spaces, Separation properties, Local Bases, linear mappings, Locally compact spaces and their finite dimensionality, Metrization of TVS, Boundedness and Continuity of Linear operators.

Semi-norms and local convexity. Minkowski functional Normability of a topological vector space. Quotient spaces.

Baire's category theorem. Banach Steinhaus theorem. Hahn-Banach theorems and their applications. Dual spaces, Open mapping theorem, Closed graph theorem in topological vector spaces.

Weak topology, Weak\* topology, Reflexivity, Banach – Alaoglu theorem, Extreme Points, Krein-Milman theorem.

**Books Recommended:**

1. W. Rudin, Functional Analysis, THM Edition
2. H.H. Schaefer, Topological Vector spaces, Macmillan, N.Y.
3. G. Kothe, Topological vector spaces, Vol. I. Springer, N.Y
4. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons
5. B. V. Limaye, Functional Analysis, Wiley Eastern
6. V. K. Krishnan, Text Book of Functional Analysis, Prentice Hall
7. A. H. Siddiqui, Functional Analysis with Applications, Tata McGraw Hill
8. L.V. Kantorovich and G.P. Akilov, Functional Analysis, Pergamon Press
9. N. Dunford & J.T.Schwartz, Linear operators - I, Interscience Wiley
10. K. Yosida, Functional Analysis, Springer – Verlag

**MMO 41 (P2) : Advanced Functional Analysis-II**

**Full Marks : 50 : Credit: 4**

Hilbert spaces, Orthogonal complements and Direct Sums, Projection Operators, Orthogonal Sets and Sequences. Total Orthonormal sets and sequences. Isomorphism of Hilbert spaces. Reflexivity of Hilbert spaces.

Sesquilinear form, Riesz representation theorem of sesquilinear form on Hilbert spaces. Adjoint operators, Lax-Milgram theorem.

Unitary operators, Normal Operators, Positive operators. Spectral Theory. Properties of spectrum of bounded linear operators in a normed linear space. Spectral mapping theorem, spectral radius, Properties of spectrum of bounded self-adjoint operators over Hilbert spaces.

Spectral theory in Banach algebra.

**Books Recommended:**

1. W. Rudin, Functional Analysis, THM Edition
2. H.H. Schaefer, Topological Vector spaces, Macmillan, N.Y.
3. G. Kothe, Topological vector spaces, Vol. 1. Springer, N.Y
4. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons
5. B. V. Limaye, Functional Analysis, Wiley Eastern
6. V. K. Krishnan, Text Book of Functional Analysis, Prentice Hall
7. A. H. Siddiqui, Functional Analysis with Applications, Tata McGraw Hill
8. L.V. Kantorovich and G.P. Akilov, Functional Analysis, Pergamon Press
9. N. Dunford & J.T.Schwartz, Linear operators - I, Interscience Wiley
10. K. Yosida, Functional Analysis, Springer – Verlag

**MMO 31 (P3) : Advanced Real Analysis-I**

**Full Marks : 50 : Credit: 4**

Differentiation of real functions. Dini derivatives. Monotonicity theorems. Vitali's covering theorem, Differentiability of monotone functions. Convex functions and their properties.

Absolutely continuous functions and singular functions. Characterization of AC functions. Cantor set and Cantor function. Indefinite Lebesgue integral. Fundamental theorem of Lebesgue integral.

The Henstock Integral: Definition and properties, Saks-Henstock Lemma

Baire classification of functions. Properties of functions of Baire class 1.

**Books Recommended:**

1. A.M. Bruckner, Differentiation of real functions, CRM Monograph Series AMS 1994.
2. Andrew M. Bruckner, Judith B. Bruckner And Brian S. Thomson, REAL ANALYSIS, PRENTICE -HALL INTERNATIONAL, INC., 1997.
3. C. Goffman, Real functions, Holt, Rinehart and Winston, New York, 1953.
4. J.D. DePree and C.W. Swartz, Introduction to real analysis, John Wiley & Sons, Inc., New York, 1988.
5. Russel A. Gordon, The Integrals of Lebesgue, Denjoy, Perron, and Henstock, Vol. 4, American Mathematical Society, 1994.



**MMO 41 (P3) : Advanced Real Analysis-II**

**Full Marks : 50 : Credit: 4**

Algebra and  $\sigma$ - algebra of sets. Borel sets.  $F_\sigma$  and  $G_\delta$ - sets. Countably additive set function. Measure on  $\sigma$ -algebra. Pre-Measure, Measurable Cover, Outer measure and measurability. Complete measure and completion of a measure. Regular outer measure. Metric Outer Measure, Lebesgue Stieltjes Measure.

Measurable function, approximation of measurable functions by simple functions. Egoroff's theorem. Lusin's theorem. Convergence in measure.

Integrals of simple function. Integrals of measurable functions. Properties of integral and integrable functions. Monotone convergence theorem. Fatou's lemma and dominated convergence theorem. Absolute Continuity, Radon-Nikodym Theorem.

**Books Recommended:**

1. A.M. Bruckner, Differentiation of real functions, CRM Monograph Series AMS 1994.
15. Andrew M. Bruckner, Judith B. Bruckner And Brian S. Thomson, REAL ANALYSIS, PRENTICE – HALL INTERNATIONAL, INC., 1997.
16. C. Goffman, Real functions, Holt, Rinehart and Winston, New York, 1953.
17. J.D. DePree and C.W. Swartz, Introduction to real analysis, John Wiley & Sons, Inc., New York, 1988.
18. Russel A. Gordon, The Integrals of Lebesgue, Denjoy, Perron, and Henstock, Vol. 4, American Mathematical Society, 1994.

**MMO 31 (P4): Advanced Topology - I**

**Full Marks : 50 : Credit: 4**

Nets, Filters - their convergence and cluster points, relation with compactness and Hausdorffness, Subnets, subfilters, Nets associated with filters and filters associated with nets, ultra-filter and compactness.

Local Connectedness: Path connectedness. Different types of compactness such as compactness, (B-W) compactness, countable compactness, sequential compactness and relations among them, Paracompactness, local compactness, Uryshon's characterization of normality and Tietze extension theorem.

Product space of a family of topological spaces : Projection mapping, Productive properties of compactness, connectedness, local connectedness,  $T_0$ ,  $T_1$ ,  $T_2$ , regularity and complete regularity. Embedding lemma,

Compactification – One point compactification, Stone-Cech compactification, Hausdorff compactification.

**Books Recommended:**

1. S. T. Hu, Introduction to General Topology, Holden-Day Inc.
33. K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd.
34. Dugungi, Topology, Prentice Hall, New Delhi.
35. G Simmons, Introduction to Topology and Modern Analysis
36. W. Thorn, Topological Structure.
37. W. J. Pervin, Introduction to General Topology, Academic Press.
38. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill

**MMO 41 (P4): Advanced Topology - II**

**Full Marks : 50 : Credit: 4**

Metrizability, Uryshon's metrization theorem, Nagata-Smirnov metrization theorem.

Uniformity, Uniform spaces, Uniform topology, Uniform continuity, Uniform isomorphism and uniform invariants, Uniformity and associated family of pseudometrics, Uniformizability and complete regularity, Compactness and uniform continuity, Completeness, Total boundedness and compactness.

Neighbourhood systems of identity, separation axioms, uniform structure, subgroups, quotient groups, locally compact groups, continuous and open homomorphism, Dual groups of locally compact Abelian groups, Dual groups of compact and discrete groups.

**Books Recommended:**

1. S. T. Hu, Introduction to General Topology, Holden-Day Inc.
2. K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd.
3. Dugungi, Topology, Prentice Hall, New Delhi.
4. G Simmons, Introduction to Topology and Modern Analysis
5. W. Thorn, Topological Structure.
6. W. J. Pervin, Introduction to General Topology, Academic Press.
7. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill

**MMO 31 (P5): Algebraic Coding Theory-I**

**Full Marks : 50 : Credit: 4**

**Basic Concepts:** Block codes and linear codes, repetition codes, nearest neighbor decoding, syndrome decoding, requisite basic ideas in probability, Shannons theorem (without proof).

**Linear and nonlinear codes:** Binary Hamming codes, dual of a code, constructing codes by various operations, simplex codes, Hadamard matrices and codes constructed from Hadamard and conference matrices, Plotkin bound and various other bounds, Gilbert-Varshamov bound.

**Reed-Muller codes:** First order Reed-Muller codes, RM code of order  $r$ , Decoding and Encoding using the algebra of finite field with characteristic two.

**Books Recommended:**

1. S. Ling and C. Xing, Coding Theory: A First Course, Cambridge University Press, 2004.
2. J. H. van Lint, Introduction to Coding Theory, Springer, 1999.
3. L. R. Vermani, Elements of Algebraic Coding Theory (Chapman and Hall Mathematics).
4. Steven Roman : Coding and Information Theory (Springer Verlag).
5. W. C. Huffman and V. Pless, Fundamentals of Error Correcting Codes, Cambridge University Press, 2003.

**MMO 41 (P5): Algebraic Coding Theory-II**

**Full Marks : 50 : Credit: 4**

**Perfect codes:** Weight enumerators, Kratchouwk polynomials, Lloyds theorem, Binary Golay codes, connections with Steiner systems.

**Cyclic codes:** The generator and the check polynomial, zeros of a cyclic code, the idempotent generators, BCH codes, Reed-Solomon codes, Quadratic residue codes, generalized RM codes, concepts of quasi-cyclic codes.

**Goppa codes:** The minimum distance of Goppa codes. Generalized BCH codes. Decoding of Goppa codes and their asymptotic behavior.

**Codes over  $Z_4$ :** Quaternary codes over  $Z_4$ , binary codes derived from such codes, Galois rings over  $Z_4$ , Cyclic codes over  $Z_4$ , Quasi-cyclic codes over  $Z_4$ .

**Books Recommended:**

1. S. Ling and C. Xing, Coding Theory: A First Course, Cambridge University Press, 2004.
2. J. H. van Lint, Introduction to Coding Theory, Springer, 1999.
3. L.R. Vermani, Elements of Algebraic Coding Theory (Chapman and Hall Mathematics).
4. Steven Roman : Coding and Information Theory (Springer Verlag).

**MMO 31 (P6): Algebraic Topology -I****Full Marks : 50 : Credit: 4**

Fundamental group, homotopy of maps between topological spaces, homotopy equivalence, contractible and simply connected spaces, fundamental groups of  $S_1$ , and  $S_1 \times S_1$  etc.

Calculation of fundamental group using Van Kampen's theorem, fundamental groups of a topological group, Brouwer's fixed point theorem, fundamental theorem of algebra, vector fields on planer sets, Frobenius theorem for matrices.

Covering spaces, unique path lifting theorem, covering homotopy theorems, group of covering transformations, criterion of lifting of maps in terms of fundamental groups, universal covering, its existence, special cases of manifolds and topological groups.

**Books Recommended:**

1. James R. Munkres, Topology-A first Course, Prentice Hall of India Pvt. Ltd.
2. Marwin J.Greenberg and J.R. Harper- Algebraic Topology-A first Course, Addison-Wesely Publishing Co.
3. C. R. Bredon - Geometry and Topology (Springer)
4. E. H. Spanier-Algebraic Topology (Springer)

**MMO 41 (P6): Algebraic Topology -II****Full Marks : 50 : Credit: 4**

Singular homology, reduced homology, Eilenberg Steenrod axioms of homology (no proof for homotopy invariance axiom, excision axiom and exact sequence axiom) and their application, relation between fundamental group and first homology.

Calculation of homology of  $S_n$ , Brouwer's fixed point theorem for  $f: E_n \rightarrow E_n$ , application spheres, vector fields, Mayer-Vietoris sequence (without proof) and its applications.

Mayer Vietoris sequence (with proof ) and its application to calculation of homology of graphs, torus and compact surface of genus  $g$ , collared pairs, construction of spaces by attaching of cells, spherical complexes with examples of  $S^n$ ,  $r$ -leaved rose, torus,  $RP^n$

Singular cohomology modules, Kronecker product, connecting homomorphism, contrafunctoriality of singular cohomology modules, naturality of connecting homomorphism, exact cohomology sequence of pair, homotopy invariance, excision properties, cohomology of a point. Mayer Vietoris sequence and its application in computation of cohomology of  $S^n$ ,  $RP^n$  torus, compact surface of genus  $g$  and non-orientable compact surface.

**Books Recommended:**

1. James R. Munkres, Topology-A first Course, Prentice Hall of India Pvt. Ltd.
39. Marwin J.Greenberg and J.R. Harper Algebraic Topology-A first Course, Addison-Wesely Publishing Co.
40. C. R. Bredon - Geometry and Topology (Springer)
41. E. H. Spanier-Algebraic Topology (Springer)

**MMO 31 (P7): Fuzzy Mathematics -I**

**Full Marks : 50 : Credit: 4**

Fuzzy sets: Basic concepts,  $\alpha$ -cuts, Properties of  $\alpha$ -cuts, Representation of a fuzzy set in terms of  $\alpha$ -cuts. Extension principle for fuzzy sets, Image and preimage of fuzzy sets under a mapping. Operations on fuzzy sets, Algebra of fuzzy sets, t-norm and t-conorm.

Fuzzy Arithmetic: Fuzzy real numbers, Representation of a fuzzy real number in terms of a family of nested intervals. Arithmetic operations on fuzzy real numbers, Lattice of fuzzy real numbers. Fuzzy equations.

Fuzzy graphs. Fuzzy relations, Composition of fuzzy relations, Transitive closure, Fuzzy preorder relation, Similitude relations. Antisymmetry, Fuzzy order relations, Dissimilitude relations, Resemblance relations, Fuzzy equivalence relations

Multivalued logics, Fuzzy propositions, Fuzzy quantifier, Linguistic hedges, Inference from – conditional fuzzy propositions, conditional and qualified fuzzy propositions, quantified fuzzy propositions

**Books Recommended:**

1. A. Kaufmann, Introduction to the theory of fuzzy subsets, (Academic Press)
42. G. J. Klir and B. Byan, Fuzzy Sets and Fuzzy logic, Theory and Applications (Prentice Hall of India)
43. S. C. Cheng, J. N. Mordeson and Y. Yandong, Lecture Notes in Fuzzy Mathematics and Computer Science, Centre for Research in Fuzzy Mathematics, Creighton University, U.S.A
44. L. Y. Ming and L. M. Kang, Fuzzy Topology (World Scientific).



**MMO 41 (P7): Fuzzy Mathematics -II**

**Full Marks : 50 : Credit: 4**

Fuzzy subgroups, fuzzy normal subgroups, Quotient fuzzy groups, Homomorphisms and Isomorphisms. Fuzzy subrings, Fuzzy ideals, Quotient fuzzy rings, Homomorphisms and Isomorphisms.

Fuzzy topological space, Multiple choice principle and neighbourhood structure, Continuous mappings, Subspaces, Product spaces, connectedness in fuzzy topological spaces, Separation properties in fuzzy topological spaces-Quasi  $T_0$ , Sub  $T_0$ ,  $T_0$ ,  $T_1$ ,  $T_2$ , Regularity. Different types of compactness in fuzzy topological spaces.

**Books Recommended:**

1. A. Kaufmann, Introduction to the theory of fuzzy subsets, (Academic Press)
12. G. J. Klir and B. Byan, Fuzzy Sets and Fuzzy logic, Theory and Applications(Prentice Hall of India)
13. S. C. Cheng, J. N. Mordeson and Y. Yandong, Lecture Notes in Fuzzy Mathematics and Computer Science, Centre for Research in Fuzzy Mathematics, Creighton University, U.S.A
14. L. Y. Ming and L. M. Kang, Fuzzy Topology (World Scientific).

**MMO 31 (P8): Lie Theory of Ordinary and Partial Differential Equations Full Marks: 50 : Credit: 4**

**Unit-I** : Lie Group of Transformations and infinitesimal Transformations:

Introduction, Lie Group of Transformations, Infinitesimal Transformations, Point Transformations and Extended Transformations (Prolongations), Multi-parameter Lie Group of Transformations and Lie Algebras, Mappings of curves and surfaces, Local Transformations.

**Unit-II** : Ordinary Differential Equations:

Elementary Examples, First-Order ODEs, Invariance of Second and Higher Order ODEs Under Point Symmetries, Reduction of Order of ODEs under multi-parameter Lie Group of Point Transformations.

**Unit-III** : Invariance of a PDE:

Introduction, Determining Equations for symmetries of a  $k^{\text{th}}$ -Order PDE, Invariance of Scalar PDE, Elementary Examples

**Unit-IV**: Invariant Solution of PDEs :

Invariant Solutions, Example, Invariance for a system of PDEs  
Determining Equations for symmetries of a system of PDEs, Examples,

**Unit-V**: Application to Boundary Value Problems. Formulation of Invariance of a Boundary Value Problem for a Scalar PDE, Incomplete Invariance for a Linear Scalar PDE, Incomplete Invariance for a Linear system of PDEs,

**Books Recommended:**

**Text Book:**

1. Symmetry and integration methods for differential equations, G W Bluman and S C Anco Springer 2002

**Reference Books:**

2. Application of Lie Groups to Differential Equations, P. J. Olver, Springer, 2000
3. Elementary Lie Group analysis and Ordinary differential equations N H Ibragimov, John Wiley & Sons 1999
4. Differential Equations: Their Solution Using Symmetries, H. Stephani Camb. Univ. Press 1989
5. Symmetry Analysis of Differential Equations With Mathematica, G Baumann, Springer (Telos), 2000.

**MMO 41 (P11): Weak Formulation of Elliptic Partial Differential Equations**

**Full Marks: 50 : Credit: 4**

**Unit-1: Distributions(17 Lectures)**

Test function space, Examples of test functions, Its topology, Distributions as continuous linear functionals, Examples of distributions, Operations on distributions: Derivation, Localization, Support, Characterization of distributions with single point support, Order of a distribution, Distributions with compact support, Multiplication by a smooth function, Product rule, Direct product and convolution product of distributions, Tempered distributions, Fourier analysis and synthesis of tempered distribution, Plancherel- Parseval identity, Riemann-Lebesgue lemma, Convolution of distributions, Approximate identity, Approximation of a distribution by smooth functions, Support theorem of convolution, Interplay between convolution and Fourier transform.

**Unit-2:Sobolev Spaces(17 Lectures)**

Definition of Sobolev Spaces, Hilbert space structure, Examples of elements, Approximation by smooth functions in the interior, Regularity of Domain, Extension of operator, Approximation by smooth functions up to the boundary.

Calculus operations: product rule, chain rule, change of variables, Fourier characterization, Dual space, Trace on the boundary, Green's formulae, Poincare' inequality, Rellich compactness lemma.

**Unit-3:Lax-Milgram Lemma and Application(6 Lectures)**

Lax-Milgram Lemma, Application to boundary value problems, various examples; Homogeneous and Inhomogeneous Dirichlet and Neumann boundary conditions.

**Books Recommended:**

1. H. Brezis, Functional Analysis, Sobolev spaces and PDE, Springer, 2011
2. S. Kesavan, Topics in Functional Analysis and Application, New Age International, 2003
3. D. Gilbarg & N. Trudinger, Elliptic PDE of second order, Springer, 1998
4. W. Rudin, Functional Analysis, TMH, 1991
5. A. Friedman, Theory of Distribution.

**MMO 31 (P9): Mathematical Logic -I****Full Marks : 50 : Credit: 4**

Formal axiomatic development, D.T and other derived rules such as Gen. Rule, Rule  $\exists$ , Rule C, Equivalence and replacement theorems, rule for change of bound variables constructing proofs of theorems. Consistency. Completeness theorem. Compactness theorem.

First order theory with equality: Equality axioms, Soundness, Completeness, Consistency of first order theory with equality. Lowenheim-Skolem theorem.

Examples of first order theory with special emphasis on formal number theory. Completeness. Categoricity, decidability, Godels incompleteness theorem.

**Books Recommended:**

1. E. Mendelson --- Introduction to Mathematical Logic, 5<sup>th</sup> edition, N.Y., Van Nostrand, 1997.
2. G.Hughes, M.J.Cresswell --- A New Introduction to Modal Logic, Routledge, 1996.
3. Fagin, Halpern, Mose, Vardi --- Reasoning about knowledge, MIT press, 1995.
4. L.Bolc, P.Borowik --- Many valued Logics, Theoretical foundations, Springer Verlag, 1992.
5. A. Margaris --- First order mathematical logic, Dover Publications, N.Y., 1990.
6. B.F.Chellas --- Modal Logic, an introduction, Cambridge University Press, 1980.
7. D.Dubois, H.Prade --- Fuzzy sets and systems – Theory and applications, Academic Press, N.Y.,1980.
8. J. Bridge --- Beginning Model theory, Oxford University Press, 1977.
9. C.C.Chang, H.J.Kaisler --- Model Theory, North Holland, 1973.
10. N. Rescher --- Many valued Logic, N.Y., McGraw Hill, 1969.

**MMO 41 (P8): Mathematical Logic -II**

**Full Marks : 50 : Credit: 4**

Many valued Logics: Informal discussion on various many valued (finite and infinite) logics truth tables validity  
Lukasiewicz 3-valued systems: a formal axiomatic study with soundness and completeness theorems.

Application of infinite valued logics in  $[0,1]$  to the development of fuzzy set theory. Calculi with t-norms and s-norms  
Introduction to approximate reasoning (basically with examples)

Modal logics: Modal propositional systems T, S<sub>4</sub>, B, S<sub>5</sub> Axioms, Kripke semantics, soundness and completeness theorems  
for the modal systems. Decidability. Logic of knowledge.

**Books Recommended:**

1. E. Mendelson --- Introduction to Mathematical Logic, 5<sup>th</sup> edition, N.Y., Van Nostrand, 1997.
11. G.Hughes, M.J.Cresswell --- A New Introduction to Modal Logic, Routledge, 1996.
12. Fagin, Halpern, Mose, Vardi --- Reasoning about knowledge, MIT press, 1995.
13. L.Bolc, P.Borowik --- Many valued Logics, Theoretical foundations, Springer Verlag, 1992.
14. A. Margaris --- First order mathematical logic, Dover Publications, N.Y., 1990.
15. B.F.Chellas --- Modal Logic, an introduction, Cambridge University Press, 1980.
16. D.Dubois, H.Prade --- Fuzzy sets and systems – Theory and applications, Academic Press, N.Y.,1980.
17. J. Bridge --- Beginning Model theory, Oxford University Press, 1977.
18. C.C.Chang, H.J.Kaisler --- Model Theory, North Holland, 1973.
19. N. Rescher --- Many valued Logic, N.Y., McGraw Hill, 1969.

**MMO31 (P10): Rings and Modules-I (Commutative Rings and Modules) Full Marks : 50 : Credit: 4**

Modules over a ring with identity, Sub modules, Operations on sub modules. Quotient Modules and module homomorphisms.

Direct products and direct sums, External direct sums and internal direct sums.

Free modules, Universal mapping properties and existence of free modules. Rings with invariant basis number, infinite dimensional vector spaces.

Tensor Products of modules, Universal Property of the tensor product, Restriction and Extension of Scalars.

Categories of modules, functors, properties of morphisms. Exact sequences and split short exact sequences. Hom and tensor product as functors and their properties. Equivalent categories and adjoint functors.

Projective and injective modules, injective modules and the functor  $\text{Hom}(-, M)$ , projective modules and the functor  $\text{Hom}(M, -)$ .

**Books Recommended:**

1. Bland, P. E., Rings and their modules, De Guyter, 2011.
2. Atiyah, M., MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. Lang, S., Algebra, Addison-Wesley, 1993.
4. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag.
4. Hungerford, T.W., Algebra, Springer.
5. Jacobson, N., Basic Algebra, II, Hindusthan Publishing Corporation, India.
6. Curtis, C.W., Reiner, I., Representation Theory of Finite Groups and Associated Algebras, Wiley-Interscience, NY.

**MMO41 (P9): Rings and Modules-II (Structure of Rings and Modules) Full Marks : 50 : Credit: 4**

Chain conditions on rings and modules. Artinian rings and Noetherian rings, Artinian module and Noetherian modules.

Jacobson radical of a ring, J-semisimple rings. Prime radical, prime ideals, prime rings and semiprime rings. Radicals and chain conditions.

Subdirect product of rings. Every ring is a subdirect product of subdirectly irreducible rings. Every Boolean ring is a subdirect product of copies of the two element field. Every semiprime ring is a subdirect product of prime rings.

Primitive ring and density. Every J-semisimple ring is a subdirect product of primitive rings. Jacobson density theorem.

Semisimple rings, Wedderburn-Artin Theory.

**Books Recommended:**

1. Bland, P. E., Rings and their modules, De Gruyter, 2011.
2. Atiyah, M., MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. Lang, S., Algebra, Addison-Wesley, 1993.
4. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag.
5. Hungerford, T.W., Algebra, Springer.
6. Jacobson, N., Basic Algebra, II, Hindustan Publishing Corporation, India.
7. Curtis, C.W., Reiner, I., Representation Theory of Finite Groups and Associated Algebras, Wiley-Interscience, NY.

**MMO 31 (P11): Theory of Computation-I****Full Marks : 50 : Credit: 4**

Resolution in Predicate Calculus: Prenex normal form, Skolemization, semantic tree, Herbrand's theorem, substitution and unification, unification algorithm, resolution techniques for first order language. Completeness of resolution technique.

Mathematical machines: Sequential machines without output ---- the transition function. Tree representation ---- connectedness. Morphisms ---- congruence relation. Quotient machines. Product machines. Sequential machines with output ---- Mealy machine, Moore machine. Automaton ---- behaviour. State minimization problem. Regular sets. Transition system and Regular Expressions. Algorithm for the construction of a minimal state deterministic finite automaton from a given regular expression. Synthesis theorem. Kleene's theorem. Decomposition of machines - algorithm. Definite events and Probabilistic machines. Definite machines. Stochastic matrices. Cut-point. Isolated cut-point. Actual probabilistic machines

**Books Recommended:**

1. H.R.Lewis, C.H.Papadimitriou --- Elements of the theory of Computation; Prentice Hall(India); 1999.
2. K.L.P.Mishra, N.Chandrasekharan --- Theory of Computer Science (Automata, Languages and Computation); Prentice Hall (India); 1999.
3. J.E.Hopcroft, J.D.Ullman --- Introduction to automata theory, languages and computation; Narosa Publishing House, India, 1987.
4. G.E.Revesz --- Introduction to formal languages; McGraw-Hill, NY, 1986.
5. M.D.Davis, E.J.Weyuker --- Computability, complexity and languages; fundamentals of theoretical computer science; Academic Press, NY, 1983.
6. R.A.Kowalski --- Logic for problem-solving, North-Holland, Amsterdam, 1979.
7. M.A.Harrison --- Introduction to formal language theory; Addison-Wesley, Reading, 1978.
8. Z. Manna --- Mathematical theory of Computation; McGraw-Hill, Kogakusha, Tokyo, 1974.
9. C.L.Chang, R.C.T.Lee --- Symbolic logic and mechanical theorem proving, Aca. Pr., NY, 1973.
10. R.Y.Kain --- Automata theory: machines and languages; McGraw-Hill, NY, 1972.
11. M.Davis --- Computability and unsolvability; McGraw-Hill, NY, 1958.



**MMO 41 (P10): Theory of Computation-II****Full Marks : 50 : Credit: 4**

Grammars and Formal Languages: definitions, basic concepts and examples. Classifications of languages and their relationship. Language and automata. Regular Grammar. Pumping Lemma. --- Applications. Context-Free Languages. Context-Free Grammars. Ambiguity in Context-Free Grammars. Pumping Lemma for Context-Free Languages. Decision algorithm for Context-Free Languages. Push-down automata and Context-Free languages. Determinism and Parsing. Top-down and Bottom-up Parsing. Context-Sensitive Languages and Linear Bounded Automata.

Turing machine and computability: Definition, basic concepts and examples of simple Turing machines. Language acceptability. Design of Turing machine. Computing with Turing machines ---- Turing computable functions. Construction of Turing machine that can compute the zero function, the successor function; perform composition, recursion and minimization. Non-deterministic Turing machines. Partial function, total function, primitive recursive function. Recursive function, partial recursive function. Turing model for computation.

Uncomputability: Unsolvability problems about Turing machines. Unsolvability problems about unrestricted grammars. Unsolvability problems about Context-Free grammars. Unsolvability Tiling problem. Thue system. Post's correspondence problem.

**Books Recommended:**

1. H.R.Lewis, C.H.Papadimitriou --- Elements of the theory of Computation; Prentice Hall(India); 1999.
2. K.L.P.Mishra, N.Chandrasekharan --- Theory of Computer Science (Automata, Languages and Computation); Prentice Hall (India); 1999.
3. J.E.Hopcroft, J.D.Ullman --- Introduction to automata theory, languages and computation; Narosa Publishing House, India, 1987.
4. G.E.Revesz --- Introduction to formal languages; McGraw-Hill, NY, 1986.
5. M.D.Davis, E.J.Weyuker --- Computability, complexity and languages; fundamentals of theoretical computer science; Academic Press, NY, 1983.
6. R.A.Kowalski --- Logic for problem-solving, North-Holland, Amsterdam, 1979.
7. M.A.Harrison --- Introduction to formal language theory; Addison-Wesley, Reading, 1978.
8. Z. Manna --- Mathematical theory of Computation; McGraw-Hill, Kogakusha, Tokyo, 1974.
9. C.L.Chang, R.C.T.Lee --- Symbolic logic and mechanical theorem proving, Aca. Pr., NY, 1973.
10. R.Y.Kain --- Automata theory: machines and languages; McGraw-Hill, NY, 1972.
11. M.Davis --- Computability and unsolvability; McGraw-Hill, NY, 1958.

## Optional Papers (Applied Stream)

**MMO31 (A1): Advanced Fluid Dynamics -I**

**Full Marks : 50 : Credit: 4**

Recapitulation: Elements of Thermodynamics Basic Concepts of Thermodynamics. Equation of State. Change of State. First Law of Thermodynamics, Internal Energy, Variables of State. Specific heats of a Gas. Entropy. Reversibility. Carnot's cycle. Second Law of Thermodynamics.

Viscous incompressible flow: Viscous fluids. Navier-Stokes equations for a Newtonian fluid.. Some solvable problems in viscous flow. Steady viscous flow in tubes. Couette flow.

Boundary-layer theory: Concept of boundary layer. Incompressible flow: Prandtl's equation of the boundary-layer. Displacement thickness, momentum thickness. Vorticity and stress components within boundary layer in 2-D motion. Separation of boundary layer from an obstacle. Blasius equation for steady 2-D flow past a flat plate. Similar solutions of the boundary layer equations: boundary layer flow past a wedge, boundary layer flow along the wall of a convergent channel, boundary layer flow past a circular cylinder.

3-D( three-dimensional) flows: Sources, sinks and doublets. Images in a rigid infinite plane. Images in solid spheres. Axisymmetric flows. Stokes stream function.

Use of Conformal transformation: Schwartz-Christoffel transformation, vortex rows, single infinite row of line vortices, Karman Vortex street.

### Books Recommended:

1. G. K Batchelor – Introduction to Fluid Dynamics, Cambridge University Press, 1967.
2. L. D. Landau and E. M. Lifscitz - Fluid Mechanics, Eng. Trans. Pergamon Press, 2<sup>nd</sup> edition 1987.
1. F. Chorlton- Text Book of Fluid Dynamics, Van Norstrand, 1967.
2. K. Oswatitsch- Gasdynamics, Eng. Trans. Academic Press,1956
3. P. Niyogi- Inviscid Gasdynmamics, Macmillan, NewDelhi 1977.
4. P. Niyogi, S.K. Chakrabartty and M.K. Laha- Introduction to Computational Fluid Dynamics, Pearson Education, Delhi 2005.
5. H. Schlichting, K. Gersten – Boundary Layer Theory, Springer-Verlag, 2004.

**MMO41 (A1): Advanced Fluid Dynamics -II****Full Marks : 50 : Credit: 4**

Magnetohydrodynamics: Electromagnetic equations for moving media, Ohm's law including Hall current, Lorentz force. MHD approximations. Stress tensor formulation of Lorentz force; Alfvén's Theorem, Alfvén waves. Equations of motion and induction, their nondimensional forms. Dimensionless parameters, Lundquist's criterion. Energy equation: Viscous and Joule dissipation, Poynting theorem. Boundary conditions.

Steady viscous incompressible flows, unidirectional flow under a transverse magnetic field, decoupling of MHD equations, Hartmann flow; Couette flow. Flow through a rectangular duct. Unsteady incompressible flows, Rayleigh's problem.

MHD waves, propagation of small disturbances, plane waves, reflection and transmission of plane harmonic waves, existence of finite amplitude MHD waves, Alfvén waves with ohmic damping, Skin effect.

Magnetohydrostatics, equilibrium configurations, Pinch effect, force-free fields, non-existence of force free field of finite extent. General solution for a force free field.

Dynamo problem, Cowling's theorem, Ferraro's law of isorotation.

**Books Recommended:**

1. G.K. Batchelor- Introduction to Fluid Dynamics, Cambridge University Press, 1967.
2. L.D. Landau and E. M. Lifschitz- Fluid Mechanics, Eng. Trans. Pergamon Press, 2nd edition"  
1987
3. F. Chorlton- Text Book of Fluid Dynamics, Van Norstrand, 1967.
- 4 J. A. Shercliff – A text book of Magnetohydrodynamics, Pergamon Press, 1965.
5. V.C. A. Ferraro and C. Plumpton- An Introduction to magneto fluid mechanics, Oxford Univ. Press, 1961.
6. A. Jeffrey – Magnetohydrodynamics, Oliver & Boyd, 1966.

### **Mathematical Models of Population Biology or Ecology**

Mathematical models: Deterministic and Stochastic. Single species population models. P-V Logistic equation. Population growth model– An age structured model.

Interactions between two species: Host-Parasite type of interactions, Competitive type of interactions. Trajectories of interactions of H-P and competitive types between two species. Effect of migration on H-P interactions. Some consequences of Lotka-Volterra equations. Generalized L-V equations. Constant of motion in the dynamical system.

Stochastic processes and need of stochastic models. Pure birth process, Pure death process, Birth and death process. Linear birth-death-immigration-emigration processes. Effects of both immigration and emigration on the dynamics of population.

Biological mechanisms responsible for "time-delay". Discrete and continuous time-delay. The single species logistic model with the effect of time-delay. Stability of equilibrium position for the logistic model with general delay function. Stability of logistic model for discrete time lag. Time-delayed H-P model together with their stability analysis.

### **Mathematical Theory of Epidemics**

Introduction; Some basic definitions. Simple epidemic model, General epidemic model. Kermack-McKendrick threshold theorem. Recurring epidemic model. A comparative study of these models. Control of an epidemic. Stochastic epidemic model without removal. Models having multiple infections. Epidemic model with multiple infections. Stochastic epidemic model with removal. Stochastic epidemic model with removal, immigration and emigration. Special discussion on the stochastic epidemic model with carriers. Simple extensions of SIR model: Different case studies --- (i) Loss of immunity, (ii) Inclusion of immigration and emigration, (iii) Immunization. SIR endemic disease model.

### **Books Recommended:**

1. J D Murray – Mathematical Biology, Springer-Verlag, Berlin (1989).
2. X Q Zhao–Dynamical Systems in Population Biology, Canadian Mathematical Society
3. R M Andersson and R M May--Infectious Diseases of Humans
4. J. N. Kapur ---Mathematical Models in Biology and Medicine, East West Press Pvt Ltd (1985)
5. R. Habermann--- Mathematical Models , Prentice Hall (1977).
6. R. W. Poole -----An Introduction to Quantitative Ecology, McGraw- Hill
7. E. C. Pielou ---An Introduction to Mathematical Ecology, Wiley, New York (1977).
8. R. Rosen ----Foundation of Mathematical Biology (vol. I& II), Academic Press
9. Mark Kot – Elements of Mathematical Ecology,Cambridge University Press (2003).

### Some Mathematical Aspects of Oscillations of the Biological Systems

Introduction; Biological Clock; Model for the circadian oscillator.

Pharmacokinetics. Mathematical models in Pharmacokinetics -Compartmental Analysis Technique. Two-compartment model -- Clinical Bromsulphalein (BSP) Test.

Basic equations for an n-compartment system. Distributions of drugs in n- compartment model for (i) given initial dose, (ii) repeated medication, (iii) constant rate of infusion and (iv) truncated infusion.

Compartment model for diabetes mellitus.

Stochastic compartment models. Drug action. Some general principles for real biological oscillations.

Cellular mechanism and genesis of Atherosclerosis.

### Arterial Biomechanics

Importance of studies on the mechanics of blood vessels. Structure and functions of blood vessels; Mechanical properties. Viscoelasticity; Linear discrete viscoelastic ( spring-dashpot ) models: Maxwell Fluid, Kelvin Solid, Kelvin Chains and Maxwell models. Creep Compliance, Relaxation Modulus. Hereditary Integrals, Stieltjes Integrals.

Constituents of blood. Structure and functions of the constituents of blood. Mechanical properties of blood.

Equations of motion applicable to blood flow. Non-Newtonian fluids - Power law, Bingham Plastic, Herschel-Bulkley and Casson fluids. Steady non- Newtonian fluid flow in a rigid circular tube. Fahraeus-Lindqvist effect. Pulsatile flow in both rigid and elastic tubes. Blood flow through arteries with mild stenosis. Shear stress on surface of the stenosis; Two-layered flow in a tube with mild stenosis.

Large deformation theory. Various forms of strain energy functions. The base vectors and metric tensors;

Green's deformation and Lagrangian strain tensors. Cylindrical model; Constitutive equations for blood vessels; Equations of motion for the vascular wall.

### Biological Diffusion and Diffusion-Reaction Models

Fick's laws of diffusion, One-dimensional diffusion model and its solution, Some solutions of two-dimensional diffusion equation, Various modifications of diffusion equation to diffusion-reaction models arising in pharmacokinetics and ecology.

Hemodialyser and dialysis of blood, Basic equations for a circular-duct and a parallel-plate dialyser, Peclet number, Sherwood number, Solutions of basic equation for a circular-duct dialyser by (i) separation of variables method and (ii) Galerkin's method. Solution for parallel-plate dialyser.

### Books Recommended:

1. J. N. Kapur ---Mathematical Models in Biology and Medicine, East West Press Pvt Ltd (1985)
2. D. A. MacDonald ----Blood Flow in Arteries, The Williams and Wilkins Company, Baltimore (1974)
3. Y. C. Fung ---Biomechanics of Soft Biological Tissues, Springer Verlag
4. R. Habermann--- Mathematical Models , Prentice Hall (1977).
5. R. W. Poole -----An Introduction to Quantitative Ecology, McGraw- Hill
- 6 E. C. Pielou ---An Introduction to Mathematical Ecology, Wiley, New York (1977)
- 7 R. Rosen ----Foundation of Mathematical Biology (vol. I& II), Academic Press
8. W Flugge- Viscoelasticity, Springer-Verlag
9. M Zamir and E L Ritman--The Physics of Pulsatile Flow

**MMO31 (A3): Computational Fluid Dynamics-I****Full Marks : 50 : Credit: 4**

Finite difference method: Treatment of model equations of parabolic, hyperbolic, elliptic types. Explicit and implicit schemes. Truncation error, consistency, convergence, stability ( Von Neumann stability analysis only) of model equations with appropriate initial and boundary conditions. Thomas algorithm. ADI method for 2-D heat conduction problem. Splitting and approximate factorization for 2-D Laplace equation. Multigrid method. First-order wave equation. Upwind scheme, consistency, CFL stability condition. First order hyperbolic system Hyperbolic conservation laws. Lax- Wendorf and McCormack schemes. Convection- diffusion equation . Stability. Finite - Volume method: Preliminary concepts, Flux computation across quadrilateral cells. Reduction of a boundary value problem to algebraic equations. Illustrative example, like solution of Dirichlet problem for 2-D Laplace equation by finite volume method

**Books Recommended:**

1. P.Niyogi, S.K.Chakraborty and M.K.Laha- Introduction to Computational Fluid Dynamics, Pearson education, Delhi 2005
2. C.A.J. Fletcher- Computational Techniques for Fluid Dynamics, Vol-I and Vol-II, Springer 1988.
3. R.Peyret and T.D.Taylor – Computational Methods for Fluid Flow, Springer 1983
4. J.F.Thompson, Z.U.A. Warsi and C.W. Martin- Numerical Grid Generation, Foundations and Applications, North Holland 1985.
5. L.D.Landau and E.M.Lifshitz, Fluid Mechanics, Trans, Pergamon Press 1989.
6. H.Schlichting and K.Gersten- Boundary – Layer Theory, 8<sup>th</sup> Ed., Springer 2000.

**MMO41 (A3): Computational Fluid Dynamics-II****Full Marks : 50 : Credit: 4**

Conservation principles of fluid dynamics. Basic equations of viscous and inviscid flow. Basic equations in conservation form. Associated typical boundary conditions for Euler and N-S equations. Lax-Wendorf and McCormack schemes for 2-D unsteady Euler equations. Grid generation using elliptic partial differential equations. Boundary-layer equations. Incompressible viscous flow field computation: Stream- function vorticity and MAC method. Turbulence modeling, Viscous compressible flow computation based on RANS using simple turbulence modeling.

**Books Recommended:**

1. P.Niyogi, S.K.Chakraborty and M.K.Laha- Introduction to Computational Fluid Dynamics, Pearson education, Delhi 2005
2. C.A.J. Fletcher- Computational Techniques for Fluid Dynamics, Vol-I and Vol-II, Springer 1988.
3. R.Peyret and T.D.Taylor – Computational Methods for Fluid Flow, Springer 1983
4. J.F.Thompson, Z.U.A. Warsi and C.W. Martin- Numerical Grid Generation, Foundations and Applications, North Holland 1985.
5. L.D.Landau and E.M.Lifshitz, Fluid Mechanics, Trans, Pergamon Press 1989.
6. H.Schlichting and K.Gersten- Boundary – Layer Theory, 8<sup>th</sup> Ed., Springer 2000.

## MMO31 (A4): Differential Equations in Ecology

Full Marks : 50 : Credit: 4

Basic concepts of continuous-time dynamical systems. Local and global bifurcations with special emphasis on Hopf-bifurcation. Centre manifold theorem. Direction of Hopf-bifurcation. Chaos. Different route to chaos. Lyapunov exponent.

Introduction to continuous time Mathematical Modeling. Non-dimensionalisation techniques.

Stability criteria of a system. Routh-Hurwitz criteria, Descartes' rule of signs. Boundedness of a system. Positiveness of a solution. Persistence of a system. Kolmogorov Analysis.

Single species differential equation models- Malthus model, Logistic model, Model with harvesting, Model with Allee effect. Insect outbreak model.

Modeling predator functional response- Holling type-I, type-II, type-III, type-IV, ratio-dependent, Beddington-DeAngelis.

Two and three species differential equation models- Predator-prey model. Food chain model. Model with one prey-two predators. Model with one predator-two preys. Model with generalist predator. Model with two preys-two predators.

Modeling two and three species continuous time models by incorporating different phenomena like imposition of a population floor, addition of refugia, omnivory, intraspecific density dependence, toxic inhibition, spatial effect, disease in prey, dispersal, predator switching, Allee effect, additional predator, additional food, harvesting of predator, disease in predator, mutualism, commensalism, parasitism, fear factor, seasonal variation, cooperation, group defense, cross predation, anti predator, etc.

Modeling of excitable systems. Chemostat Model. Tumour-growth model. Cancer model. HIV-model. Model of divorce prediction and marriage repair. Metapopulation and patch Model.

Epidemic Models (SI, SIR, SIS, SIRS, SEIR etc ).

Models with time delay (single and double delay). Derivation of a critical delay for stability of a system. Periodic solutions.

Sensitivity analysis and Numerical simulations of the models using MATLAB and MATHEMATICA. Practical applications of continuous time ecological models.

**N.B. Discussion on all the topics mentioned above will be made with reference to continuous systems by using differential equations approach.**

### Books Recommended:

1. A.D. Bazykin, A.I. Khibnik, B. Krauskopf, Nonlinear dynamics of interacting populations (World Scientific).
2. J.D. Murray, Mathematical Biology (Vol. I, II) (Springer-Verlag).
3. D.W. Jordan, P. Smith, Nonlinear Ordinary Differential Equations: An introduction to Dynamical Systems (Oxford University Press).
4. S. H. Strogatz, Nonlinear dynamics and chaos with applications to Physics, Biology, Chemistry, and Engineering (Westview Press)
5. J. Pastor, Mathematical Ecology of Populations and Ecosystems (Wiley-Blackwell).
6. J. Guckenheimer, P.J. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields (Springer).



7. B.D. Hassard, N.D.Kazarinoff, Y.H. Wan, Theory and Applications of Hopf Bifurcation (London Mathematical Society).
8. M. Kot, Elements of Mathematical Ecology (Cambridge).

## MMO41 (A4): Difference Equations in Ecology

Full Marks : 50 : Credit: 4

Basic concepts of discrete-time dynamical systems. Local and global bifurcations with special emphasis on Neimar-sacker bifurcation and flip bifurcation. Centre manifold theorem. Direction of Hopf-bifurcation. L Chaos. Different route to chaos. Lyapunov Exponent.

Introduction to Mathematical Modeling. Non-dimensionalisation techniques.

Stability criteria of a system. Jury's condition. Boundedness of a system. Positiveness of a solution. Persistence of a system. Kolmogorov Analysis.

Single species difference equation models-Malthus model, Logistic model, Model with harvesting, Model with Allee effect. Insect outbreak model.

Modeling predator functional response- Holling type-I, type-II, type-III, type-IV, ratio-dependent, Beddington – DeAngelis.

Two and three species difference equation models- Predator-prey model. Food chain model. Model with one prey-two predators. Model with one predator-two preys. Model with generalist predator. Model with two preys-two predators.

Modeling two and three species discrete time models by incorporating different phenomena like imposition of a population floor, addition of refugia, omnivory, intraspecific density dependence, toxic inhibition, spatial effect, disease in prey, dispersal, predator switching, Allee effect, additional predator, additional food, harvesting of predator, disease in predator, mutualism, commensalism, parasitism, fear factor, seasonal variation, cooperation, group defense, cross predation, anti predator, etc.

Modeling of excitable systems. Chemostat Model. Tumour-growth model. Cancer model. HIV-model. Model of divorce prediction and marriage repair. Metapopulation and patch Model.

Epidemic Models (SI,SIR, SIS, SIRS, SEIR etc ).

Models with time delay (single and double delay). Derivation of a critical delay for stability of a system. Periodic solutions.

Sensitivity analysis and Numerical simulations of the models using MATLAB and MATHEMATICA.

Practical applications of discrete time ecological models.

**N.B. All the topics listed above will be studied for discrete systems, using difference equations approach.**

### Books Recommended:

1. A.D. Bazykin, A.I. Khibnik, B. Krauskopf, Nonlinear dynamics of interacting populations (World Scientific).
2. J.D. Murray, Mathematical Biology (Vol. I, II) (Springer-Verlag).
3. D.W. Jordan, P. Smith, Nonlinear Ordinary Differential Equations: An introduction to Dynamical Systems (Oxford University Press).

4. S. H. Strogatz, Nonlinear dynamics and chaos with applications to Physics, Biology, Chemistry, and Engineering (Westview Press)
5. J. Pastor, Mathematical Ecology of Populations and Ecosystems (Wiley-Blackwell).
6. J. Guckenheimer, P.J. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields (Springer).
7. B.D. Hassard, N.D.Kazarinoff, Y.H. Wan, Theory and Applications of Hopf Bifurcation (London Mathematical Society)
8. M. Kot, Elements of Mathematical Ecology (Cambridge).

**Temporal dynamics (I)**

**1. Introduction**

**2. Phase Portraits for planar systems:** Canonical forms, Eigenvectors defining stable and unstable manifolds, Phase portraits of linear systems in the plane, Linearization and Hartman's theorem, Constructing phase plane diagrams

**3. Classical single species population models:** Population growth-An age structure model

**4. Models for Interacting Populations**

(i) Two species predator-prey population model, Classical Lotka-Volterra model, Competing populations, Symbiotic populations, Leslie-Gower model, Classical Holling-Tanner model, Other growth models; Migration / Harvesting models and optimal control theory, Model with prey switching / generalist predator / Allee effect (Boukal-Sabelis-Berec model) etc.

(ii) Local and Global Stability

(iii) Food Web

(iv) Age-dependent populations

**5. Epidemic models:** Basic epidemic models, Other classical models, Age- and stage-dependent epidemic system, Case study: various diseases, Analysis of a disease with two states

**6. Eco-epidemic systems:** Prey-diseased-predator interactions, Predator-diseased-prey interactions, Diseased competing species models, Eco-epidemics models of symbiotic communities (Disease effects on the symbiotic system, Disease control by use of a symbiotic species)

**Books Recommended:**

- 1.J.D. Murray, Mathematical Biology (Volumes I, II), Springer-Verlag, Berlin, 1993
- 2.R.M. May, Stability and complexity in model ecosystems, Princeton University Press, 2001
- 3.D.W. Jordan, P. Smith, Nonlinear Ordinary Differential Equations: An introduction to Dynamical Systems, Vol. 2, Oxford University Press, 1999
- 4.Lawrence Perko, Differential equations and dynamical systems, Springer (Third Edition), 2001
- 5.Steven H. Strogatz, Nonlinear dynamics and chaos with applications to Physics, Biology, Chemistry, and Engineering, Persues Books Publishing, 1994
- 6.A.D. Bazykin, A.I. Khibnik, B. Krauskopf, Nonlinear dynamics of interacting populations, World Scientific Publishing Company Incorporated, Singapore, 1998
7. J. Burdon, Diseases and plant population biology, Cambridge University Press, 1987
8. J.N. Kapur, Mathematical models in biology and medicine, East-West Press Pvt. Ltd., 1985
9. F.R. Giordano, M.D. Weir, W.P. Fox, Mathematical Modelling (Third edition)
10. B.D. Hassard, N.D. Kazarinoff, Y.H. Wan, Theory and application of Hopf-bifurcation, Cambridge University Press, Cambridge, 1981

**Temporal dynamics (II)**

1. **Three-dimensional autonomous systems and chaos:** Linear systems and canonical forms, Nonlinear systems and stability, The Róssler system and chaos, The Lorenz equations, The Belousov-Zhabotinski reaction
2. **Three species population model:** Competitive food chain model, Mathematical model formulation, Stability and bifurcation analysis, System behaviour around boundary equilibria, System behaviour near the coexistence equilibrium, Numerical simulation by MATLAB and MAPLE
3. **Modelling with delay differential equations:** Introduction, Different models using delay differential equations, Linear stability analysis, Miscellaneous examples
4. **Modelling with stochastic differential equations:** Introduction, Some stochastic models, Equilibria and local stability analysis, Biological implications

**Spatiotemporal dynamics and pattern formation**

1. **Spatial aspect:** Diffusion as a paradigm
4. **Diffusion processes:** Diffusion and random motion, Nonlinear diffusion models, Reaction-diffusion (Fisher's equation, Travelling wave solutions, Perturbation solution, Stability of travelling waves, Nagumo's equation), Advection-diffusion (Burgers' equation, Travelling wave solutions, Initial value problem), Asymptotic solution to Burgers' equation, Reaction-diffusion systems
5. **Instabilities and dissipative structures:** Reaction-diffusion (Turing) mechanisms, Turing patterns through diffusion-driven instability, Turing space, Mode selection and the dispersion relation, Differential flow instability, Ecological example (semiarid vegetation patterns etc.)
6. **Patterns in the wake of invasion:** Invasion in a predator-prey system, Dynamical stability of an unstable equilibrium, Patterns in a competing species community
7. **Patchy invasion:** Allee effect, biological control and 1D patterns of species invasion, Invasion and control in the 2D case, Biological control through infectious diseases

**Books Recommended:**

1. J.D. Murray, Mathematical Biology (Volumes I, II), Springer-Verlag, Berlin, 1993
2. R.M. May, Stability and complexity in model ecosystems, Princeton University Press, 2001
3. A. Okubo, Diffusion and ecological problems: mathematical models, Springer-Verlag, Berlin (FRG), 1980
4. A. Okubo, S.A. Levin, Diffusion and ecological problems: modern perspective, Springer-Verlag, 2001
5. D.W. Jordan, P. Smith, Nonlinear Ordinary Differential Equations: An introduction to Dynamical Systems, Vol. 2, Oxford University Press, 1999
6. Lawrence Perko, Differential equations and dynamical systems, Springer (Third Edition), 2001

7. Steven H. Strogatz, Nonlinear dynamics and chaos with applications to Physics, Biology, Chemistry, and Engineering, Persues Books Publishing, 1994
8. Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, Academic Press, Boston, 1993
9. D.S. Jones, M. Plank, B.D. Sleeman, Differential equations and mathematical biology, Chapman and Hall (CRC), 2009
10. R.S. Cantrell, C. Cosner, Spatial ecology via reaction-diffusion equations, Wiley, England, 2003
11. H. Meinhardt, Models of biological pattern formation, Academic Press, London, 1982
12. Horst Malchow, Sergei V. Petrovskii, Ezio Venturino, Spatiotemporal patterns in ecology and epidemiology: Theory, models, simulations. CRC Mathematical and Computational Biology Series. CRC Press, Boca Raton, 2008

**MMO 31 (A6): Dynamical Meteorology****Full Marks : 50 : Credit: 4**

Thermodynamics of dry and wet air. Eulerian equations of continuity and of motion for atmospheric motion and their forms in spherical coordinates rotating with the Earth. Circulation and vorticity theorems. Energy and entropy equations. Scale analysis of basic equations for mid-latitude synoptic systems. Quasi-static equations for adiabatic frictionless motions. Geostrophic wind, . thermal wind equation, inertial, cyclostrophic and gradient motions. Basic equations for mean with Reynolds' stresses, The mixing length theory, Ekman layer equation. Richardson's Criterion for persistence of turbulence. Static stability on a resting earth. Dry and wet adiabatic lapse rate. Oscillations and waves in a compressible atmosphere under gravity. Application to Lee waves. Stability of zonal currents for horizontal non-divergent motion. Rossby wave in barotropic and baroclinic currents. Numerical Forecasting. Quasi-geostrophic equations. Applications to one parameter and two-parameter atmospheric models. Cyclones and anticyclones. The general circulation. Tropical motion systems. Monsoons.

**Books Recommended:**

1. James R. Holton, An introduction to Dynamic Meteorology Academic Press, New York, 1992.
2. George J. Haltiner, Frank L. Martin --- Dynamical and Physical Meteorology, McGraw Hill, 1957.
3. I.P.Bazarov, Thermodynamics, Pergamon Press, Oxford, 1964.
4. V.M. Kamenkovich, Fundamentals of Ocean Dynamics Elsevier Scientific Publ. Company, 1977.
5. Adrian E.Gill, Atmosphere -Ocean Dynamics, Academic Press, London, 1982

**MMO 41 (A6) Dynamical Oceanography****Full Marks : 50 : Credit: 4**

Thermodynamics of sea water as a two-component system. Gibbs relation. Gibbs-Duhem relation. Conditions of thermodynamic equilibrium of finite volume of sea water, Vaisala frequency, Equations of conservation of mass, Boundary conditions at the ocean surface. Equation of motion of sea water. Equation of energy and entropy transfer. Linearised equations for small amplitude wave motion in spherical coordinates. Boussinesq approximation. f3 -plane equations for motion of seawater. Equation of energy for linear wave motion. Eigen value problems for determination of free linear waves on a sphere. Gravity waves in an exponentially stratified fluid. Planetary waves. Theory of tides. Reynolds' equation of mean motion and boundary conditions at the mean ocean surface. Quasi- static approximation. Geostrophic motion. Pure Drift current. Ekman's theory of wind-driven current in homogeneous ocean. Ekman boundary layer. Western boundary flow. Two-dimensional and three-dimensional models of ocean currents. Simple linear model of a thermocline.

**Books Recommended:**

1. James R. Holton, An introduction to Dynamic Meteorology Academic Press, New York, 1992.
2. George J. Haltiner, Frank L. Martin --- Dynamical and Physical Meteorology, McGraw Hill, 1957.
3. I.P.Bazarov, Thermodynamics, Prgaman Press, Oxford, 1964.
4. V.M. Kamenkovich, Fundamentals of Ocean Dynamics Elsevier Scientific Publ. Company, 1977.
5. Adrian E.Gill, Atmosphere -Ocean Dynamics, Academic Press, London, 1982



**MMO 31 (A7): Lie Theory of Ordinary and Partial Differential Equations Full Marks: 50 : Credit: 4**

**Unit-I** : Lie Group of Transformations and infinitesimal Transformations:

Introduction, Lie Group of Transformations, Infinitesimal Transformations, Point Transformations and Extended Transformations (Prolongations), Multi-parameter Lie Group of Transformations and Lie Algebras, Mappings of curves and surfaces, Local Transformations.

**Unit-II** : Ordinary Differential Equations:

Elementary Examples, First-Order ODEs, Invariance of Second and Higher Order ODEs Under Point Symmetries, Reduction of Order of ODEs under multi-parameter Lie Group of Point Transformations.

**Unit-III** : Invariance of a PDE:

Introduction, Determining Equations for symmetries of a  $k^{\text{th}}$ -Order PDE, Invariance of Scalar PDE, Elementary Examples

**Unit-IV**: Invariant Solution of PDEs :

Invariant Solutions, Example, Invariance for a system of PDEs  
Determining Equations for symmetries of a system of PDEs, Examples,

**Unit-V**: Application to Boundary Value Problems. Formulation of Invariance of a Boundary Value Problem for a Scalar PDE, Incomplete Invariance for a Linear Scalar PDE, Incomplete Invariance for a Linear system of PDEs,

**Books Recommended:**

**Text Book:**

1. Symmetry and integration methods for differential equations, G W Bluman and S C Anco Springer 2002

**Reference Books:**

2. Application of Lie Groups to Differential Equations, P. J. Olver, Springer, 2000
3. Elementary Lie Group analysis and Ordinary differential equations N H Ibragimov, John Wiley & Sons 1999
4. Differential Equations: Their Solution Using Symmetries, H. Stephani Camb. Univ. Press 1989
5. Symmetry Analysis of Differential Equations With Mathematica, G Baumann, Springer (Telos), 2000.

**MMO 41 (A13): Weak Formulation of Elliptic Partial Differential Equations Full Marks: 50 : Credit: 4**

**Unit-1: Distributions(17 Lectures)**

Test function space, Examples of test functions, Its topology, Distributions as continuous linear functionals, Examples of distributions, Operations on distributions: Derivation, Localization, Support, Characterization of distributions with single point support, Order of a distribution, Distributions with compact support, Multiplication by a smooth function, Product rule, Direct product and convolution product of distributions, Tempered distributions, Fourier analysis and synthesis of tempered distribution, Plancherel- Parseval identity, Riemann-Lebesgue lemma, Convolution of distributions, Approximate identity, Approximation of a distribution by smooth functions, Support theorem of convolution, Interplay between convolution and Fourier transform.

**Unit-2:Sobolev Spaces(17 Lectures)**

Definition of Sobolev Spaces, Hilbert space structure, Examples of elements, , Approximation by smooth functions in the interior, Regularity of Domain, Extension of operator, Approximation by smooth functions up to the boundary.

Calculus operations: product rule, chain rule, change of variables, Fourier characterization, Dual space, Trace on the boundary, Green's formulae, Poincar inequality, Rellich compactness lemma.

**Unit-3:Lax-Milgram Lemma and Application(6 Lectures)**

Lax-Milgram Lemma, Application to boundary value problems, various examples; Homogeneous and Inhomogeneous Dirichlet and Neumann boundary conditions.

**Books Recommended:**

1. H. Brezis, Functional Analysis, Sobolev spaces and PDE, Springer, 2011
2. S. Kesavan, Topics in Functional Analysis and Application, New Age International, 2003
3. D. Gilbarg & N. Trudinger, Elliptic PDE of second order, Springer, 1998
4. W. Rudin, Functional Analysis, TMH, 1991
5. A. Friedman, Theory of Distribution.

**(Plasma dynamics preliminaries)****1. Introduction**

1.1 Basic properties of plasmas: \*[4 lectures]

- Definition, occurrence and importance of plasmas, Debye shielding
- Quasi-neutrality, plasma parameter, plasma frequency, Larmor orbits (basics)
- Non-ideal plasmas

1.2 Thermonuclear fusion and plasma confinement: [2 lecture]

- Fusion reactions, Conditions for fusion
- Overviews: magnetic confinement fusion (MCF) and tokamaks, inertial confinement fusion (ICF)

**2. Elements of plasma dynamics** [6 lectures]

2.1 Single particle motion

- Cyclotron motion, Electrodynamics equations for a conducting fluid (Maxwell's equations, Conservation of electric charge, Generalized Ohm's law etc.)
- Guiding center drifts:  $\mathbf{E} \times \mathbf{B}$ , curvature and gradient
- Magnetic moment, adiabatic invariants, magnetic mirrors

3. **Plasma as fluids:** [6 lectures]

Introduction

The fluid equation of motion (Continuity equation, Momentum balance equation, Equations of state, Two-fluid equations, Plasma resistivity)

3. **Waves in plasmas** [8 lectures]

3.1 Plasma oscillations, Langmuir waves, ion-acoustic waves

3.2 Electromagnetic waves (parallel and perpendicular to  $\mathbf{B}_0$ ), Cutoffs and resonances

4. **Kinetic plasma theory** [6 lectures]

4.1 Vlasov and Boltzmann equations, Obtaining fluid equations from Boltzmann equation, From the two-fluid to MHD description of plasmas

4.2 Langmuir waves, resonant particles and trapping, Landau damping

5. **Nonlinear effects in plasmas:** [8 lectures]

Introduction, The Sagdeev potential, Derivation of KdV equation for ion-acoustic waves, Soliton solution in one dimension, Elementary ideas about the ponderomotive force and parametric instability.

*\*All numbers of lectures are approximate.*

**Recommended books:**

1. Fundamentals of Plasma Physics, J. A. Bittencourt, Springer-Verlag New York, 2008.
2. Introduction to Plasma Physics and Controlled Fusion, F. F. Chen, Plenum Press, New York, Second Edition, Vol. 1, 1984.
3. Introduction to Plasma Physics, R. J. Goldston and P. H. Rutherford, IOP Publishing, Bristol & Philadelphia, 1995.

**MMO 41 (A7): Magnetohydrodynamics-II**

**Full Marks: 50 : Credit: 4**

**(Basic magnetohydrodynamics)**

- 1. The MHD model:** [10 lectures]  
1.1 Basic concepts of MHD, MHD equations (Two-fluid MHD, Resistive MHD, Ideal MHD, Conservation form, Generalized Ohm's law)  
1.2 The convective derivative, Characteristic lengths and frequencies, MHD validity and assumptions  
1.3 Magnetic flux: Flux tubes, Conservation of magnetic flux  
1.4 Conservation of energy, Magnetic helicity  
1.5 Reduced MHD equations, Magnetic viscosity and Reynold's number, Frozen-in magnetic fields
- 2. MHD equilibrium:** [10 lectures]  
2.1 MHD equilibrium equations  
2.2 Magnetic pressure: The concept of beta  
2.3 Plasma confinement in a magnetic field  
2.4 Resistive MHD  
2.5 The Rayleigh-Taylor instability, Two stream instability, The Weibel instability
- 3. Linear MHD waves:** Introduction, Alfvén waves, Magnetosonic waves [4 lectures]
- 4. High-frequency EM waves in a magnetized plasma:** [10 lectures]  
4.1 Propagation parallel and perpendicular to the magnetic field—LCP and RCP waves; Ordinary and Extraordinary waves; Resonance and Reflection points  
4.2 Some special wave phenomena in plasmas: Atmospheric Whistlers, Faraday Rotation
- 5. Nonlinear MHD waves:** [6 lectures]  
Solitary waves, Shocks in dissipative magneto-plasmas.

***\*All numbers of lectures are approximate.***

**Recommended Books:**

1. T. G. Cowling, Magnetohydrodynamics, Interscience Publishers, New York, 1957.
2. J. A. Shercliff, A Text Book of Magnetohydrodynamics, Pergamon Press, Oxford, 1965.
3. S. I. Pai, Magnetohydrodynamics and Plasma Dynamics, Springer Verlag, New York, 1962.
4. K. R. Cramer and S. I. Pai, Magnetofluid Dynamics for Engineers and Applied Physicists, McGraw Hill, New York, 1973.
5. Fundamentals of Plasma Physics, J. A. Bittencourt, Springer-Verlag New York, 2008.
6. Introduction to Plasma Physics and Controlled Fusion, F. F. Chen, Plenum Press, New York, Second Edition, Vol. 1, 1984.

**MMO31 (A9): Mathematical Pharmacology-I      Full Marks : 50 : Credit: 4**

**Cell Surface Receptor/Ligand Binding Fundamentals:**

Specific binding; simple monovalent binding-constant ligand concentration, ligand depletion effect; cooperativity; multiple receptor states-simple one-step binding; non-interconverting receptor classes; interconverting receptor classes and their applications; basic ternary complex model and applications.

**The Endocytic Cycle:**

Simple kinetic models of endosomal sorting-ligand motion, receptor motion; whole-cell kinetic model and applications; kinetic model of sorting that predicts ligand effects-monovalent receptor-monovalent ligand, monovalent receptor-bivalent ligand.

**Diffusivity and the Mechanism of Mass Transport:**

Fick's law of binary diffusion (molecular mass transport); theory of diffusion in binary liquids, gases at low density, colloidal suspensions, polymers; mass and molar concentration; diffusive mass and molar fluxes; convective mass and molar fluxes.

General form of conservation equations-for finite volumes, for points and for interfaces; Conservation of chemical species; illustrations with simple examples; simplification based on time scales; solution of diffusion equation with/without elimination or generation-in rectangular and curvilinear coordinates and applications.

**Recommended books:**

1. Lauffenburger, D.A. and Linderman, J., 1993. Receptors: models for binding, trafficking, and signaling. Oxford University Press.
2. Ciechanover, A., Schwartz, A.L. and Lodish, H.L., 1985. Sorting and Recycling of Cell Surface Receptors and Endocytosed Ligands. In Mechanisms of Receptor Regulation (pp. 225-253). Springer US.
3. Evans, E.A., Skalak, R. and Weinbaum, S., 1980. Mechanics and thermodynamics of biomembranes.
4. Ramkrishna, D. and Amundson, N.R., 1985. Linear operator methods in chemical engineering with applications to transport and chemical reaction systems. Prentice Hall.
5. Segel, L.A., 1980. Mathematical models in molecular cellular biology. Cambridge University Press.
6. Wells, J.W. and Hulme, E.C., 1992. Receptor-Ligand Interactions: A Practical Approach. Oxford, IRL Press.
7. Limbird, L.E., 2012. Cell Surface Receptors: A Short Course on Theory and Methods. Springer Science & Business Media.
8. Linderman, J.J. and Lauffenburger, D.A., 2013. Receptor/ligand sorting along the endocytic pathway (Vol. 78). Springer Science & Business Media.
9. Pastan, I. and Willingham, M.C., 1985. The pathway of endocytosis. In Endocytosis (pp. 1-44). Springer US.
10. Deen, W.M., 1998. Analysis of Transport Phenomena. Indian Edition. Oxford University Press.
11. Bird, R.B., Stewart, W.E. and Lightfoot, E.N., 2007. Transport phenomena. John Wiley & Sons.

**Pharmacokinetics of Drug Distribution:**

Compartmental models-one-compartment models; two-compartment models;two-compartment models with absorption; physiological models-flow-limited compartments; drug binding and metabolism in compartmental models; application of physiological models such as microvascular mixing in tissue compartments.

Chemical reaction; diffusion with a homogeneous and heterogeneous chemical reaction; diffusion and chemical reaction inside a porous catalyst; multi-component system; concentration distribution with more than one independent variable-time dependent diffusion. Concepts of free and bound drug; irreversible, reversible, saturable and non-saturable binding of drug; effective diffusion coefficient and tortuosity; diffusion with binding in tissues.

Drug permeation through biological barriers-through lipid membranes, through porous membranes; drug transport by fluid motion-simple examples.

**Controlled Drug Delivery System:**

Reservoir and transdermal delivery system-diffusion through planar membranes, diffusion through cylindrical membranes; matrix delivery system.

Case studies: design of a controlled delivery system for T-20, design of a controlled delivery system from implanted devices-matrix type delivery device and drug-eluting stent.

**Books Recommended:**

1. Lauffenburger, D.A. and Linderman, J., 1993. Receptors: models for binding, trafficking, and signaling. Oxford University Press.
2. Ciechanover, A., Schwartz, A.L. and Lodish, H.L., 1985. Sorting and Recycling of Cell Surface Receptors and Endocytosed Ligands. In Mechanisms of Receptor Regulation (pp. 225-253). Springer US.
3. Evans, E.A., Skalak, R. and Weinbaum, S., 1980. Mechanics and thermodynamics of biomembranes.
4. Ramkrishna, D. and Amundson, N.R., 1985. Linear operator methods in chemical engineering with applications to transport and chemical reaction systems. Prentice Hall.
5. Segel, L.A., 1980. Mathematical models in molecular cellular biology. Cambridge University Press.
6. Wells, J.W. and Hulme, E.C., 1992. Receptor-Ligand Interactions: A Practical Approach. Oxford, IRL Press.
7. Limbird, L.E., 2012. Cell Surface Receptors: A Short Course on Theory and Methods. Springer Science & Business Media.
8. Linderman, J.J. and Lauffenburger, D.A., 2013. Receptor/ligand sorting along the endocytic pathway (Vol. 78). Springer Science & Business Media.
9. Pastan, I. and Willingham, M.C., 1985. The pathway of endocytosis. In Endocytosis (pp. 1-44). Springer US.
10. Deen, W.M., 1998. Analysis of Transport Phenomena. Indian Edition. Oxford University Press.
11. Bird, R.B., Stewart, W.E. and Lightfoot, E.N., 2007. Transport phenomena. John Wiley & Sons.
12. Saltzman, W.M., 2001. Drug delivery: engineering principles for drug therapy. Oxford University Press.
13. Murray J D., 2003. Mathematical Biology (vol.-II), Springer.

**MMO31 (A10): Nonlinear Differential Equations - I**

**Full Marks : 50 : Credit: 4**

1<sup>st</sup> order PDEs: Conservation equation, applied to traffic problems, nonlinear traffic flow problems.  
Perturbation Techniques: Method of multiple scale, Mathieu Equation, van-dar Pol Oscillator, Diffusion-advection equation, KBJ Approximation,  
An Introduction to nonlinear waves, Solitary waves and Solitons, Shock Waves.  
2<sup>nd</sup> order PDEs: Burger's Equations: Historical background, derivation of Burger Equation in Fluids and Plasmas. Properties of Burger Equation. Shock wave and other solutions of Burger's Equation. Cole-Hopf Transformation.  
KdV Equation: Historical background, derivation of KdV Equation in Fluids and Plasmas. Properties of KdV Equation. One Soliton and Other solution of KdV Equation  
NLS Equation: Historical background, derivation of NLS Equation in Plasmas. Properties and Different types of solutions of NLS Equation.

**Recommended Books:**

1. Perturbation Methods (Cambridge Texts in Applied Mathematics) by E J Hinch (Cambridge University Press 1991)
2. Solitons, Nonlinear Evolution Equations and Inverse Scattering by Mark J. Ablowitz & P. A. Clarkson (Cambridge University Press 1991)
3. Partial Differential Equations and Solitary Waves Theory by Abdul-Majid Wazwaz (Springer Science & Business Media 2010)
4. Solitons: An Introduction by P. G. Drazin & R. S. Johnson (Cambridge University Press 1989)
5. Nonlinear waves, solitons and chaos by E. Infeld & G. Rowlands (Cambridge University Press 1990)

**MMO41 (A9): Nonlinear Differential Equations - II**

**Full Marks : 50 : Credit: 4**

Solution of different nonlinear differential equations by Tanh, tanh-coth, sin-cosine, Exponential function methods.

Hirota's Bilinear Method : Introduction, D operator, Bilinear form of KdV, KP like equations, Hirota's perturbation and multi soliton solutions of KdV, KP, MKdV, MKP, SG equations.

Inverse Scattering Technique: Introduction, Conservation Laws, Exact solution by inverse scattering, LAX approach, Solution of some standard nonlinear equation.

To study periodic waves, quasi-periodicity and chaos from KdV, KP and ZK equations by using the theory of planer dynamical system.

Introduction to Modified KdV, KP, ZK equations in planer and non-planer geometry. Transformation of different equations to KdV Equation.

**Recommended Books:**

1. Perturbation Methods (Cambridge Texts in Applied Mathematics) by E J Hinch (Cambridge University Press 1991)
2. Solitons, Nonlinear Evolution Equations and Inverse Scattering by Mark J. Ablowitz & P. A. Clarkson (Cambridge University Press 1991)
3. Partial Differential Equations and Solitary Waves Theory by Abdul-Majid Wazwaz (Springer Science & Business Media 2010)
4. Solitons: An Introduction by P. G. Drazin & R. S. Johnson (Cambridge University Press 1989)
5. Nonlinear waves, solitons and chaos by E. Infeld & G. Rowlands (Cambridge University Press 1990)



**MO31 (A11): Quantum Mechanics -I****Full Marks : 50 : Credit: 4**

Old Quantum Theory: (a) Black body radiation-Planck's Hypothesis, (b) Electromagnetic radiation --- the photoelectric effect and the Compton effect (c) De Broglie's waves, (d) Bohr's postulates and discrete levels. Dynamical variables and observable: (a) Linear operators, (b) Eigenvalues and eigenfunctions (c) Commutation relations, (d) Angular momentum operators - the eigenvalue equation for  $L^2$ , (e) Observable - the general physical interpretation (f) Dirac's bra and ket notation. The Physical Postulates: (a) The correspondence

principle (b) The complementary principle, (c) The uncertainty principle- limitations on experiment, (e) Packets in space and time.

Schrodinger's wave equation: (a) The fundamental properties - statistical interpretation, normalization, (b) The current density, (c) Energy eigenfunctions. Discrete Eigenvalues: Bound states = (a) One dimensional motion well potential, Linear harmonic oscillator, (b) The hydrogen atom - (i) Separation in spherical polar coordinates reduced mass, asymptotic behaviour, energy levels, wavefunctions.

Approximate Method for Bound States: (a) Stationary Perturbation Theory

**Books Recommended:**

1. Quantum Mechanics - L I Schiff, 3rd edition, McGraw Hill, 1968.
2. The Principles of Quantum Mechanics - P.A.M. Dirac, 4th edition. Clarendon, Oxford, 1958.
3. Introduction to Quantum Mechanics - L. Pauling and E.B. Wilson, Jr. Mc Graw Hill (Dover) 1985
4. Quantum Mechanics - E. Merzbacher, 2nd edition, Wiley, 1970.
5. Quantum Mechanics - L. E. Ballentine, Prentice Hall 1990
6. Atomic and Molecular Physics – B. H. Bransden and C. J. Joachain
7. The Theory of Atomic Collisions – N. F. Mott and H. S. W. Massey (Oxford)

**MMO41 (A10): Quantum Mechanics-II****Full Marks : 50 : Credit: 4**

Continuous Eigen values: Collision Theory - (a) One dimensional square potential barrier-asymptotic behaviour, scattering coefficients, (b) Scattering by spherically symmetric potentials -asymptotic behaviour, scattering cross section, method of partial waves.

Identity of Particles and spin: (a) The spin operators, Pauli spin matrices, spin and Statistics (b) The Exclusion principle.

Approximate Methods for Bound States: (a) The Variation method, (b) Ground state energy of helium.

Approximate Methods in collision Theory : (a) Born approximation, (b) Partial-wave method.

Relativistic Wave Equation: (a) Derivation of the Dirac equation - Dirac matrices, charge and current densities. (b) Theory of positrons.

**Books Recommended:**

1. Quantum Mechanics - L I Schiff, 3rd edition, McGraw Hill, 1968.
2. The Principles of Quantum Mechanics - P.A.M. Dirac, 4th edition. Clarendon, Oxford, 1958.
3. Introduction to Quantum Mechanics - L. Pauling and E.B. Wilson, Jr. Mc Graw Hill (Dover) 1985
4. Quantum Mechanics - E. Merzbacher, 2nd edition, Wiley, 1970.
5. Quantum Mechanics - L. E. Ballentine, Prentice Hall 1990
6. Atomic and Molecular Physics – B. H. Bransden and C. J. Joachain
7. The Theory of Atomic Collisions – N. F. Mott and H. S. W. Massey (Oxford)

**MMO 31 (A12): Solid Mechanics-I****Full Marks : 50 : Credit: 4**

Analysis of Strain: Finite and small strain tensor. Rotation Components. State of strain at a point, Principle Strains and Strain invariants. Strain compatibility equations. Determination of displacement from prescribed strain components. Cesaro's formulas. Multiply connected regions. Dislocation problems. Problems.

Analysis of Stress: Stress Vector and Stress tensor. State of stress at a point. Principal stresses. Stress Invariants. Stress equations of motion and equilibrium. Surface conditions. Stress functions. Maximum Normal and shear stresses. Mohr's Diagram. Problems.

Orthogonal Curvilinear Co-ordinates: Strains and Rotations in orthogonal curvilinear co-ordinates, Stress equations of motion and equilibrium in orthogonal curvilinear co-ordinates. Special cases in various co-ordinate systems. Problems: Lamé's problems of a closed spherical shell and solid sphere.

Stress –Strain Relations: Generalized Hooke's Law. Anisotropic bodies. Elastic Symmetry. Isotropic bodies. Elastic Moduli. First and Second Laws of Thermodynamics. Strain Energy Function. Clapeyron's Theorem. Displacement equations of equilibrium and motion in different co-ordinates systems. Beltrami-Michell compatibility equations.

Fundamental Boundary Value Problems of Elastostatic and Elastodynamics: Uniqueness of solutions. Saint-Venant's Principle. Saint- Venant's Semi-inverse method.

Two-Space Elastostatic Problems of Linear Elasticity: Plane strain, plane stress and generalized plane stress. Airy's stress function. Compatibility Equation. Rotating disks and other simple problems. Complex representation of a 2-space biharmonic function. Complex representation of boundary conditions. Kolosov-Muskhelishvile formulae for displacements and stresses in terms of two complex potentials. Arbitrariness in the definitions and structures of the complex potentials in multiconnected regions. First and second fundamental boundary value problems of plane elasticity. Uniqueness of solutions. Use of conformal mapping. Solutions of simple problems under uni-axial or bi-axial tension at infinity of an infinite plate/region. Infinite plate or region with an elliptic hole. Equations of Muskhelishvile and their solutions in the case of simple problems. Solutions of boundary value problems for a half plane. Concentrated normal load on a straight boundary.

Saint-Venant's Problem: Torsion of rods of arbitrary uniform cross-section. Torsion of a bar of varying circular section. Complex torsion function. Solution of torsion problem using conformal mapping. Flexure of isotropic beams fixed at one end by terminal transverse loads at the other end. Centre of flexure simple problems.

3-Space Elastostatic Problems of Linear Elasticity: Displacement functions. Displacement potentials. Love's displacement function. Integral of the equations of equilibrium in terms of Galerkin Vector, Boussinesq-Papkovich-Neuber solution of the equations of equilibrium. Concentrated force at a point of an infinite body. Concentrated normal load on the plane boundary of a semi-infinite body. Distributed normal load on the boundary of a semi-infinite body. Equilibrium of an elastic sphere under no forces: Special solutions of the displacement equations of equilibrium in terms of spherical harmonics.

Variational Methods: Reciprocal theorem of Betti and Rayleigh, Theorem of Minimum Potential Energy.

**Books Recommended:**

1. Love, A.E.H., A treatise on the mathematical theory of elasticity (Dover, 1944).
2. Sokolnikoff, L.S., Mathematical theory of elasticity (McGraw Hill, 1956)
3. Coraff, F.Karl; Wave motion in elastic solids (Clarendon press, Oxford, 1975)
4. Waslay, Richard J : Stress wave propagation in solids (Marcel Dekker lin, 1973)
5. Hill, R: The mathematical theory of plasticity (Oxford, 1956)
6. Hoffman, O & Sachs, G: Introduction to the theory of plasticity for engineers (McGraw Hill, 1953)
7. Kachanov, LM: Fundamental of the theory of plasticity, MIR Pub.,1974

**MMO41 (A11): Solid Mechanics-II****Full Marks : 50 : Credit: 4**

Vibration of elastic solids: The general problem of vibrations, Uniqueness of solution. Free and forced vibrations of elastic solids. Conjugate property of normal functions. Roots of frequency equation. Longitudinal and flexural vibrations of a rod. Torsional vibration of a circular cylinder. Radial vibration of sphere and circular cylinder.

Propagation of waves in elastic solid media: Waves of dilatation and waves of distortion Motion of a surface of discontinuity, kinematical and dynamical conditions, velocity of waves in isotropic and anisotropic media plane waves and spherical waves. Surface waves. Rayleigh waves and Love waves Reflexion and refraction of elastic waves.

Plastic deformation and flow equations: Elastic and plastic deformation. The stress-strain curve. Strain hardening. The ideal plastic body. The criterion of yielding. Tresca-Saint Venant condition and von Mises condition of yielding plastic flow: general relations, Prandtl Reuss equations, Saint Venant –von Mises theory of plasticity. Elasto-plastic problems: Bending of a prismatic bar Circular cylinder under torsion, spherical shell under internal pressure, Cylindrical tube under pressure. Inverse method for the solution of elastoplastic torsion problem (Sokolovsky's method). Slip lines, properties of slip lines, equations of slip lines under plane strain condition.

**Books Recommended:**

1. Love, A.E.H., A treatise on the mathematical theory of elasticity (Dover, 1944).
2. Sokolnikoff, L.S., Mathematical theory of elasticity (McGraw Hill, 1956)
3. Coraff, F.Karl; Wave motion in elastic solids (Clarendon press, Oxford, 1975)
4. Waslay, Richard J : Stress wave propagation in solids (Marcel Dekker Inc, 1973)
5. Hill, R: The mathematical theory of plasticity (Oxford, 1956)
6. Hoffman, O & Sachs, G: Introduction to the theory of plasticity for engineers (McGraw Hill, 1953)
7. Kachanov, LM: Fundamental of the theory of plasticity, MIR Pub.,1974

**MMO 31 (A13): Theory of Computation-I****Full Marks : 50 : Credit: 4**

Resolution in Predicate Calculus: Prenex normal form, Skolemization, semantic tree, Herbrand's theorem, substitution and unification, unification algorithm, resolution techniques for first order language. Completeness of resolution technique.

Mathematical machines: Sequential machines without output ---- the transition function. Tree representation ---- connectedness. Morphisms ---- congruence relation. Quotient machines. Product machines. Sequential machines with output ---- Mealy machine, Moore machine. Automaton ---- behaviour. State minimization problem. Regular sets. Transition system and Regular Expressions. Algorithm for the construction of a minimal state deterministic finite automaton from a given regular expression. Synthesis theorem. Kleene's theorem. Decomposition of machines - algorithm. Definite events and Probabilistic machines. Definite machines. Stochastic matrices. Cut-point. Isolated cut-point. Actual probabilistic machines

**Books Recommended:**

1. H.R.Lewis, C.H.Papadimitriou --- Elements of the theory of Computation; Prentice Hall(India); 1999.
2. K.L.P.Mishra, N.Chandrasekharan --- Theory of Computer Science (Automata, Languages and Computation); Prentice Hall (India); 1999.
3. J.E.Hopcroft, J.D.Ullman --- Introduction to automata theory, languages and computation; Narosa Publishing House, India, 1987.
4. G.E.Revesz --- Introduction to formal languages; McGraw-Hill, NY, 1986.
5. M.D.Davis, E.J.Weyuker --- Computability, complexity and languages; fundamentals of theoretical computer science; Academic Press, NY, 1983.
6. R.A.Kowalski --- Logic for problem-solving, North-Holland, Amsterdam, 1979.
7. M.A.Harrison --- Introduction to formal language theory; Addison-Wesley, Reading, 1978.
8. Z. Manna --- Mathematical theory of Computation; McGraw-Hill, Kogakusha, Tokyo, 1974.
9. C.L.Chang, R.C.T.Lee --- Symbolic logic and mechanical theorem proving, Aca. Pr., NY, 1973.
10. R.Y.Kain --- Automata theory: machines and languages; McGraw-Hill, NY, 1972.
11. M.Davis --- Computability and unsolvability; McGraw-Hill, NY, 1958.

**MMO 41 (A12): Theory of Computation-II****Full Marks : 50 : Credit: 4**

Grammars and Formal Languages: definitions, basic concepts and examples. Classifications of languages and their relationship. Language and automata. Regular Grammar. Pumping Lemma. --- Applications. Context-Free Languages. Context-Free Grammars. Ambiguity in Context-Free Grammars. Pumping Lemma for Context-Free Languages. Decision algorithm for Context-Free Languages. Push-down automata and Context-Free languages. Determinism and Parsing. Top-down and Bottom-up Parsing. Context-Sensitive Languages and Linear Bounded Automata.

Turing machine and computability: Definition, basic concepts and examples of simple Turing machines. Language acceptability. Design of Turing machine. Computing with Turing machines ---- Turing computable functions. Construction of Turing machine that can compute the zero function, the successor function; perform composition, recursion and minimization. Non-deterministic Turing machines. Partial function, total function, primitive recursive function. Recursive function, partial recursive function. Turing model for computation.

Uncomputability: Unsolvability problems about Turing machines. Unsolvability problems about unrestricted grammars. Unsolvability problems about Context-Free grammars. Unsolvability Tiling problem. Thue system. Post's correspondence problem.

**Books Recommended:**

1. H.R.Lewis, C.H.Papadimitriou --- Elements of the theory of Computation; Prentice Hall(India); 1999.
2. K.L.P.Mishra, N.Chandrasekharan --- Theory of Computer Science (Automata, Languages and Computation); Prentice Hall (India); 1999.
3. J.E.Hopcroft, J.D.Ullman --- Introduction to automata theory, languages and computation; Narosa Publishing House, India, 1987.
4. G.E.Revesz --- Introduction to formal languages; McGraw-Hill, NY, 1986.
5. M.D.Davis, E.J.Weyuker --- Computability, complexity and languages; fundamentals of theoretical computer science; Academic Press, NY, 1983.
6. R.A.Kowalski --- Logic for problem-solving, North-Holland, Amsterdam, 1979.
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8. Z. Manna --- Mathematical theory of Computation; McGraw-Hill, Kogakusha, Tokyo, 1974.
9. C.L.Chang, R.C.T.Lee --- Symbolic logic and mechanical theorem proving, Aca. Pr., NY, 1973.
10. R.Y.Kain --- Automata theory: machines and languages; McGraw-Hill, NY, 1972.
11. M.Davis --- Computability and unsolvability; McGraw-Hill, NY, 1958.