Department of Mathematics
VISVA-BHARATI

Ph. D. Course Work

Paper – I (100 marks)
Paper – II (100 marks)
Paper – III (100 marks)

Paper – I (Marks: 100)

Unit –I: Computer Applications (25 marks)
Unit- II: Research Methodology for Mathematics (25 marks)
Unit- III & Unit – IV are to be chosen by the candidate from the following topics of 25 marks each:

MPM-01	**: Computational Linear Algebra**
MPM-02	**: Mathematical Modelling**
MPM-03	**: Approximation of Continuous Systems**
MPM-04	**: Numerical ODEs**
MPM-05	**: Numerical PDEs**
MPM-06	**: Methods of Applied Mathematics I**
MPM-07	**: Method of Applied Mathematics II**
MPM-08	**: Dynamical System I**
MPM-09	**: Dynamical System II**
MPM-10	**: Theory of Stochastic Processes**
MPM-11	**: Non-linear Ordinary Differential Equations in**
MPM-12	**: Complex Analysis**
MPM-13	**: Universal Algebra**
MPM-14	**: Fuzzy Set and Fuzzy Logic**
MPM-15	**: Topology**
MPM-16	**: Introduction to Semigroups**
MPM-17	**: Generalised Integrals**
MPM-18	**: Functional Analysis**

<u>Paper – II (Marks : 100)</u> One special paper is to be chosen from the following list

MPS-01 : Methods for Integrating Nonlinear Differential Equations
MPS-02 : Mathematical Biology – I
MPS-03 : Mathematical Biology - II
MPS-04 : Functions of Complex Variables
MPS-05 : Fuzzy Functional Analysis
MPS-06 : Algebraic Theory of Semigroups
MPS-07 : Fuzzy Algebra
MPS-08 : Fluid Mechanics
MPS-09 : Advanced Real Analysis II
MPS-10 : Plasma Dynamics
MPS-11 : Meteorology of Air Pollution
MPS-12 : Computational Fluid Dynamics
MPS-13 : Introduction to Topological Dynamics
MPS-14 : Uncertainty – Theory of Possibility, Plausibility and Usuality
MPS-15 : Formal Language, Grammar and Automata
MPS-16 : Quantum Scattering Theory

Paper – III (Marks : 100)

Dissertation work is to be carried out under the guidance of a faculty member of the Department to be set by the Board of Studies in Mathematics. Topic is to be chosen in order to help pursue further research.

Research Methodology for Mathematics

I. Concepts

Research --- Definition: Commonsense, Dictionary, Technical.
Definition --- Concepts of Construct, Postulate, Proposition, Thesis, Hypothesis, Law, Principle.
Definition and Dimension of a Theory, Functions and Characteristics.
Types of Theory: General Theory, Particular/ Empirical Theory and Empirical Theorizing; Cases and Their Limitations, Causal Relations.
Differences between Theory, Principle, Law and; Hypothesis, Postulate, Proposition.
Hypothesis – Formulation: *Ex-Ante/A Priori* and *Ex-post* hypothesis formulation.
Sources of Hypothesisation.
Empirical Basis of Hypothesis Formulation, Types of Errors in Formulation of Hypothesis, Testing of Hypothesis - Meaning.

II. Objectives, Structure and Design of Investigation

Selection of research problem and objective(s) of investigation;
Research Design.
Preparation of Synopsis: Statement and Explanation of Nature and Importance of Problem of investigation; Objectives of Investigation; Hypotheses; Data or Information Base --- Sources of Data; Models and Methods; Chapter Scheme and References --- Techniques of Referencing and Maintenance of Record of studies; Review of Literature: Meaning, Functions and Importance, Types and Techniques of Reviewing Literature.

III. Originality in Science Research

Originality: Meaning, Discovery, Innovation and Invention --- their meanings and implications for originality.
Role of Creativity, Intuition, Imagination, Vision and Perception.
Limitations of Social/Human phenomena and their impact on Research.

IV. Other Methods

Scientific Method and steps involved in scientific method; Methods of Induction and Deduction; Comparative and Historical Method; Experimental and Experiential Method; Observational and Participatory Method; Case Method of Research and Development of Cases.

Suggested Readings

Burns, Robert B. (2000) *Introduction to Research Methods*, Sage Publications, New Delhi.
Cooper, D. R. and Schindler, P. R.(1999) *Business Research Methods*, Tata Mcgraw Hill.
Smith, E., Thorpe, M., Hill, R. and Lowe, A. (2002) *Management Research – An Introduction*, Sage Publications, N. Delhi.
Popper, Karl R. (1968) *The Logic of Scientific Discovery*, Hutchinson of London.
Sarvavanavel, P. (2003) *Research Methodology*, Kitab Mahal, Allahabad.

MPM-01: Computational Linear Algebra:

Direct methods for linear systems: triangular equations, Gauss elimination, LU-decomposition, conditioning and finite-precision arithmetic, partial and complete pivoting, Cholesky factorisation, band matrices, QR-factorisation. Iterative methods for linear systems: Richardson, Jacobi, Gauss-Seidel, SOR; block variants; convergence criteria; Chebyshev acceleration. Symmetric eigenvalue problems: power method and variants, Jacobi's method, Householder reduction to tridiagonal form, eigenvalues of tridiagonal matrices, the QR method. Krylov subspace methods: Lanczos method; conjugate gradient method, preconditioning. Introduction to multigrid.

MPM-02 : Mathematical Modelling

Introduction: What is a Mathematical Model and what are the issues involved in developing such a model?

Modeling with Deterministic Differential Equations: Introduction to autonomous dynamical systems arising from ordinary differential equations, example models include predator-prey systems and mechanical problems; the heat equation provides an example of a partial differential equation.

Probabilistic Modeling: Review of elementary probability theory, Markov chains and stochastic matrices, random walks, heuristic derivation of the heat equation using tools from probability theory.

MPM-03: Approximation of Continuous Systems

Interpolation and approximation: Taylor’s theorem in 1D. Lagrange interpolating polynomials: error formulae; Newton’s difference formula for equally spaced nodes; Runge phenomenon; interpolating polynomials with Chebyshev nodes. Piece- wise polynomial interpolation in 1D and 2D: Hermite interpolation; 2D interpolation; 1D splines (cubic, parametric). Data fitting. Basis functions.

Integration and differentiation: Standard 1D composite methods of integration: trapezoidal rule; midpoint rule; Simpson’s rule; Gauss rules; error estimates. Multiple integrals: standard methods; Monte Carlo methods. Adaptive integration. Finite differences for approximating derivatives. Dealing with noisy data in approximate integration and differentiation.

Root finding: Scalar problems ($f(x) = 0$): bisection; regula-falsi; Newton’s method. Multi-dimensional Taylor formula. Multi-dimensional Newton method. Continuation.

Basic iterative methods. Projection methods. Krylov subspace methods. Preconditioned iteration and preconditioning techniques. Methods for nonlinear systems of equations: fixed point methods, Newton’s method, quasi□Newton methods, steepest descent techniques, homotopy and continuation methods. Numerical ODEs: Euler’s methods, Runge□Kutta Methods, multi□step method, shooting method. Numerical PDEs: Introduction to finite difference and finite element methods. Fast linear system solvers: multi-grid method.

MPM-04 : Numerical ODEs

Introduction and Basic Concepts: definitions, reduction to first order system of ODEs, Euler's method, global and local truncation errors (LTE), convergence, Taylor series sch-emes
Runge Kutta Methods: derivation of explicit schemes, stability, efficiency, vector-valued problems
Time Step and Error Control: error estimates based on two different methods, efficient use of RK schemes in error control
Linear Multistep Methods: derivation, LTE, implicit vs. explicit, linear difference equations, zero and absolute stability, stiffness, backward difference formulae
Boundary Value Problems: finite differences

MPM-05 : Numerical PDEs

Finite difference approximations : classification of PDE's; forward, backward and central dfferences, Taylor series.
Parabolic PDE's : finite difference approximation of the heat equation; local truncation error analysis; stability and convergence; multi-level and ADI schemes.
Hyperbolic PDE's : travelling wave solutions; comparison of schemes for the advection equation; second order wave equations; nonlinear conservation laws.
Elliptic PDE's: Introduction, simple finite difference scheme. Finite element method (FEM); variational formulation of the FEM; nodal basis functions and matrix elements.

MPM-06: Methods of Applied Mathematics I

1. Function spaces, completeness, square integrable functions. Orthogonal sets of vectors and the Bessel inequality. Basis and Parseval's relation. Weierstrass' theorem. Classification of orthogonal polynomials, classical orthogonal polynomials. Trigonometric series, generalized functions

2. Second order differential equations. Solutions of DEs by integral representations. Integral representations of hypergeometric functions.

3. Regular perturbations, Poincare–Lindstedt method. Singular purturbations, boundery layer problems. WKB approxinmation, asymptotic expansion of integrals.

4. Calculus of variations, necessary condition, Euler–Lagrange equations, Lagrange functions depending on higher derivatives, null Lagrange functions, lagrange function of given DE. Lagrange function with several dependent variables. Iso-perimetric problems.

MPM-07: Methods of Applied Mathematics II

1. Partial differential equation: Solution of wave equation, heat equation and Laplace equation by Integral Transform
2. Integral equations and Green's functions.Hilbert–Schmidt theory of singular integral equations.
3. Stability and bifurcation. One-dimensional problems, two-dimensional problems. Hydrodynamics stability.

MPM-08: Dynamical System I

One dimensional complex dynamics, hyperbolic dynamical systems, symbolic dynamics, strange attractors, fractals in higher dimensions. Julia sets, Mandelbrot sets, quasi□conformal mappings
Dynamical systems (discrete time): Orbits, fixed points, periodic orbits, asymptotic points and phase portraits of dynamical systems. Stability and instability of fixed points and periodic orbits. Basic ideas of chaos, chaotic functions and orbit. Symbolic dynamics — the sequence space, the shift map and its properties, symbolic dynamics for the shift map, topological conjugacy of the shift map to 'normal' maps on R. Sarkovskii's theorem — 'period 3 implies chaos'.

MPM-09: Dynamical System II

Local Bifurcations: Structural stability. Implicit function theorem. Fold, pitchfork and transcritical bifurcations in 1D. Centre Manifold Theorem. Hopf Bifurcation Theorem.
Discrete Dynamical Systems: Fixed points and bifurcations for maps. Numerical methods as maps.
Computing Fixed Points: Indirect by IVP, direct by Newton's method. Numerical Continuation, natural continuation and arc-length.
Periodics and Nonlinear Oscillations: Poincare maps, Stability of periodic orbits. Relaxation oscillators. Coupled Oscillators.
Global Bifurcations: Homoclinc and heteroclinic orbits. Idea of co-dimension. Relation to travelling waves, computation.
Conservative and Hamiltonian Systems: Definitions. Conservative forces, Hamiltonian, Symplectic. Maps and numerical methods.
Dissipative Systems: Gradient systems, Dissipativity, Global attractors.

MPM-10: Theory of Stochastic Processes

Definition and basic concepts. The random walk --- unrestricted random walk. Types of stochastic processes. Gamblers' ruin problem. Generalization of the random walk model. Markov chains --- definition, Chapman-Kolomogorov equation, equilibrium distribution, classification of states. Stochastic process in continuous time --- Poisson process.

References:

1. J.L.Doob --- Stochastic processes; John Wiley, NY, 1953.
2. M.S.Barlett --- An introduction to stochastic process, CUP, 1955.
3. Z.Brzezniak and T.Zastawniak --- Basic stochastic processes, Springer; 2005.
4. J.Medhi --- Stochastic processes, Wiley Eastern Ltd. 1983.

MPM-11: Nonlinear Ordinary Differential Equation in Complex Domain

Equations of the first order but not of first degree in complex domain:

Specification of the equations, Conditions for the absence of branch point, Discussion on the dependent variable initially infinite, Equation into which independent variable does not enter explicitly (Autonomous system), Binomial equations of arbitrary degree and their classifications, Integration of Binomial equations of all classes, Abel equation and Weierstrassian elliptic function.

Nonlinear Equations of Second Order:

Statement of the. problem, General solution as a function of the constants of integration (Nonlinear Superposition), Outline of the method of solution and their applications. The first necessary conditions for the absence of the movable singular points, The second necessary conditions for the absence of the movable singular points, Reduction to standard form and classification of equations, Tables of equations in canonical form, The Painleve equations and their solutions, The Painleve transcendent, The first Painleve transcendent: freedom from movable branch point, Freedom from movable essential singularity, The main proof in the case when absolute value of the dependent variable has a positive lower bound, Discussion of the case in which the lower bound of absolute value of dependent variable is zero, Representation of the transcendent as the quotient of two integral functions, The arbitrary constants which enter into the transcendent, Asymptotic relationship between the first Painleve transcendent and Weierstrassian elliptic function, Equations of the second order algebraic in dependent variable, General conclusion.

Analytical solutions of nonlinear ordinary differential equations by using software of computer algebras viz., Maple, Mathematica.

Refs.: Chapters XIII & XIV, Ordinary Differential Equation, E LInce

MPM-12 : Complex Analysis

Power Series : Position of the singularities, Convergence of the series and regularity of the function, Form of a power series having unit radius of convergence which diverges at two prescribed points on the unit circle and converging at all other points on the circle, Stoltz's theorem.
Dirichlet Series: Convergence, Absolute Convergence, Definition of σ_0 , $\overline{\sigma_0}$ and their relation, Convergence of the series and regularity of the function, Asymptotic behaviour.

Reference:

1.Theory of Functions, E.C.Titchmarsh, Oxford University Press, 1961.
2.Dirichlet Series, G.Hardy, Dover Publication.

MPM-13 : Universal Algebra

The poset Eq(A) of all equivalence relations on a set A is a complete lattice. Relational product of two equivalence relations; join and meet of two equivalence relations in the lattice Eq(A). The lattice Eq(A) is isomorphic to the lattice Π(A) of all partitions of A. Definition and examples of algebraic lattice; algebraic closure operators; every algebraic lattice is isomorphic to the lattice of all closed subsets of some set A with an algebraic closure operator C.
Definition and examples of algebras; subalgebras and subuniverses, every algebraic lattice is isomorphic to the lattice of all subuniverses of some algebra A.
Congruences and quotient algebras. The poset Con(A) of all congruences on an algebra is an algebraic lattice. Congruence-distributive, congruence-modular and congruence-permutable algebras. Homomorphisms and the isomorphisms theorems.
Direct products, factor congruences and directly indecomposable algebras; every finite algebra is isomorphic to a direct product of directly indecomposable algebras. Subdirect products, subdirectly irreducible algebras; every algebra A is isomorphic to a subdirect product of subdirectly irreducible algebras. Simple algebras; for a congruence ρ on an algebra A, A/ ρ is a simple algebra iff ρ is a maximal congruence on A or ρ = A × A.
Class operators I, S, H, P, P_s; varieties (define as a class of algebras which is closed under subalgebras, homomorphic images and direct products), V=HSP. Term, term algebras and free algebras; universal properties and uniqueness (upto isomorphism) of the free algebras.
Identities and Birkhoff's Theorem : a class of algebras is a variety if and only if it is an equational class.

References:

1. Burris and Sankappanavar : A Course in Universal Algebra, Springer.
2. G. Gratzer : Universal Algebra, Springer.
3. G. Gratzer : General Lattice Theory, Birkhauser Verlag.
4. G. Birkhoff : Lattice Theory, Amer. Math. Society.

MPM-14 : Fuzzy Set and Fuzzy Logic

Fuzzy set: Basic concepts --- definition of fuzzy set, a-cut, properties of a-cut, representation of a fuzzy set in terms of a-cut. Extension principle for fuzzy set. Image and pre-image of fuzzy set under a mapping. Operations on fuzzy set. Algebra of fuzzy sets. Fuzzy function --- t-norm and t-conorm.

Fuzzy Logic: Fuzzy logic in a narrow sense; Fuzzy logic in a wide sense. Concept of linguistic variables and their values --- the term sets. Modifiers and linguistic hedges. Fuzzy connectives --- their interpretations. Fuzzy rules --- quantification and qualification. Fuzzy rule-based inference --- Compositional rule of inference. Industrial applications of fuzzy logic.

References:

1. G.J.Klir, Bo Yuan --- Fuzzy sets and fuzzy logic: theory and applications, Prentice-Hall India.
2. Petr Hajek --- Metamathematics of fuzzy logic, Academic Press.

MPM-15 : Topology

Uniform Spaces: Metrization of unifonn spaces, the gage of uniformity, Cauchy nets, extension of functions, completion.
Proximity spaces: Definition of Efremovic proximity; topology induced by a proximity; Proximity mappings; Subspaces and products of proximity spaces; Proximity induced by a uniformity; Proximal convergence.

Books Recommended:
1. J. L. Kelley, General Topology, East-West Press Pvt. Ltd.
2. S.A. Naimpally & B.D. Warrack, Proximity spaces, Cambridge Univ. Press. ,

MPM-16 : Introduction to Semigroups

Definitions and examples of semigroups; semigroup of subsets of a set, semigroup of relations on a set, semigroups of partial transformations and full transformations on a set.
Subsemigroups, cyclic semigroups, ideals (left, right), bi-ideals and quasi-ideals of a semigroup, principal ideals, simple semigroups. Congruences, factor semigroups, homomorphisms on semigroups, lattice of congruences, Rees congruence. Direct and subdirect product of semigroups; every semigroup is a subdirect product of subdirectly irreducible semigroups. Bands and semilattices, a semilattice is a commutative band. Regular semigroups and inverse semigroups, a regular semigroup is inverse if and only if idempotents commute, characterization of regular semigroups by its ideals, natural partial order on inverse and regular semigroups. Orthodox semigroups. Free semigroups, free semilattices

Green's equivalences; egg-box structure: every H –class is an intersection of an L –class and an R –class. Green's equivalences on the transformation semigroups. An H – class can contain at most one idempotent. An H – class is a group if and only if it contains an idempotent. Characterizations of Green's equivalences on regular and inverse semigroups.

Band of semigroups, semilattice of semigroups, the least semilattice congruence N on a semigroup, every band is a semilattice of rectangular band, a band is rectangular if and only if it is simple, left zero band and right zero band, every rectangular band is a direct product of a left zero and a right zero band. Units and maximal subgroups. Embedding of semigroups in a group.

Ordered semigroups: Partially ordered, fully ordered and lattice ordered semigroups. The positive and negative cone. Ideals in an ordered semigroups and simple ordered semigroups. Green's equivalences on an ordered semigroup.

References:

1. Clifford and Preston : The Algebraic Theory of Semiroups (Vol. I, II), Amer. Math. Society.
2. J. M. Howie : Fundamentals of semigroup Theory, Clarendon Press, Oxford.
3. M. Petrich : Introduction to semigroups, Merrill, Columbus.
4. L. Fuchs : Partially Ordered Algebraic Systems, Pergamon Press.

MPM-17 : Generalised Integrals

Differentiation of real functions. Dini derivates. Monotonicity theorems.
Vitali's covering theorem, Differentiability of monotone functions. Convex functions and their properties.
The Perron integral: Definitions and basic properties, Comparison with Lebesgue integral and Newton integral.

MPM-18 : Functional Analysis

Compact Linear Operators on Normed linear spaces. Some geometrical structures in normed linear spaces viz. radius, Chebyshev radius, Chebyshev centre, diametral point, normal structure etc. Uniformly convex and strictly convex normed linear spaces. Some fixed point theorems in normed linear spaces (viz. Browder-Kirk, Schauder etc.).

1. E. Kreyszig, Introductory Functional Analysis with Applications, Jhon Wiley & Sons. .
2. K. Goebel, W. A. Kirk, Topic in metric fixed point theory, Cambridge University Press.
3. B. V. Limaye, Functional Analysis, Wiley Eastern Limited.

Paper-II : Marks: 100

MPS-1: Methods for Integrating Nonlinear Differential Equations

Contents: The course aims to provide a general introdution to Painleve and symmetry analysis of differential equations and will concentrate on the technical aspects of how to derive . movable sinularities, resonance and compatibility conditions, symmetries and their use to calculate exact solutions, first integrals or conservation laws of nonlinear ordinary differential equations (ODEs) and nonlinear partial differential equations (PDEs). This course contains some of the more recent developments regading nonlocal transformations between differential equations (for ODEs) and the related classifcation problems, as well as some aspects of recursion operators for Lie-Baecklund symmetries of ODEs and PDEs and integrable hierarchies of differential equation,s. Finally, computer practical classes consist of programming of salient analytical results of above mentioned procedures in the language of symbolic computation like Mathematica or Maple have been introduced to improve the calculational efficiency of the researcher.
Since the aim of this course is to address efficient methods towards the solution of nonlinear differential equations appear in wide range of applied sciences, we will not discuss the deeper mathematical significance of some of the statements for the singularity analysis and symmetry calculations. References will however be provided for the interested reader for further exploration on the subject. The course can be divided into the following main sections*:*

Lie transformation groups and Lie group generators
Prolongations and Lie algebras.
Lie point symmetries: lnvariance of the differential equations.
Symmetries of ODEs and their reductions, solutions and invariants.
Symmetries of PDEs and their reductions, solutions and invariants.
λ- symmetry of ODEs and its utility.
Painleve Analysis of ODEs.
Painleve Analysis of PDEs.
Lie-Baecklund symmetries, recursion operators and integrability.
Transformations between differential equations.
Basic principle of Mathematica and Maple softwares.
Familiarity with the library functions of Mathematica and Maple.
Use of Mathematica and Maple for calculation of symmetry, invariants, reduction and exact analytical solution, recursion operators etc. of ODEs and PDEs.

Literature:
Symmetry Methods for Differential Equations: A Beginner's Guide P. E. Hydon (Cambridge University, Press - 2000)
Symmetries and Integration Methods for Differential Equations G. W. Bluman and S. C. Anco (Springer-Verlag - 2002) .
Applications of Lie Groups to Differential Equations P. J. Olver (Springer-Verlag - 1st edition 1986 and 2nd edition 1993)
CRC Handbook of Lie Group Analysis of Differential Equations Vols. LILIII; Ed by N H Ibragimov

Ordinary Differential Equations, ELlnce, (DoverPubl., London 1956)
Painlev'e analysis of nonlinear PDE and related topics, R. Conte, (Academic Press, New York, 1989).
The Painleve Property and singularity analysis of Integrable and Nonintegrable system A Ramani, B Gramaticos and T Bountis, Physics Report Vol 180, 159-245 (1989)
The Painlev'e property, one century later, R. Conte, CRM series in mathematical " physics (Springer, New York, 1999)
Direct and inverse methods in nonlinear evolution equations, Ed. by A. Greco (Springer, Berlin, 2002)
Symmetry Analysis of Differential Equations with Mathematica, G Baumann (Springer, Telos, 2000)
Mathematical Computing: An Introduction to Programming Using Maple, Mylan Redfern and David Betounes (Telos Pr,2002)
Computer Algebra System: A New Software Toolbox for Maple, Victor Aladjov (Fultus Corporation, 2004)
Dynamical system with applications using Maple, S. Lynch (2nd Ed. Birkhauser, 2009)

MPS-2: MATHEMATICAL BIOLOGY-I

Population Dynamics and Mathematical Ecology:

Introduction to mathematical models in biology ; Deterministic and Stochastic models ; Population models ; Single species population models ; Density dependent growth ; Phase plane solution of the logistic equation ; Explicit solution of the logistic equation ; Logistic growth models with time delay ; Stability of equilibrium position for logistic model with general delay function ; Stability of logistic model for discrete time lag; Discrete one-species model with an age distribution ; Stochastic birth processes ; Stochastic death processes ; Stochastic birth and death processes ; Linear birth-death-immigration-emigration processes ; Steady-state nonlinear birth-death processes.

Introduction to two-species models ; Type of interactions between two species ---- host-parasite type and competitive type ; Trajectories of interactions of host-parasite and competitive type between two species ; Predator-prey models with time delays ; Derivation of the generalized Lotka-Volterra (L-V) equations ; Some consequences of L-V equations ; Limitations of L-V equations ; Phase plane of linear systems highlighting the behavior of the equilibrium points towards the stability of the system.

Mathematical Epidemiology:

Introduction to various terminologies relevant to epidemiology ; Basic idea of the classification of epidemic ----- simple, general and recurring epidemics ; Deterministic models without removal : A simple deterministic epidemic model ; SIS model with constant and specific time-dependent rate of infection ; Simple epidemic model with carriers. General deterministic epidemic model with removal ; Kermack-McKendrik threshold theorem ; Recurring epidemic model. Control of an epidemic ; Stochastic epidemic model without removal ; Epidemic with multiple infections ; Stochastic epidemic models with removal ; Stochastic epidemic models with removal , immigration and emigration ; Stochastic epidemic models (a) with carriers (b) with infectives and carriers ; Special discussion on the epidemic model with carriers.

Mathematical aspects of oscillations of the biological system:

Introduction to the concept of biological oscillations ; Biological clock ; A model for the circadian oscillator ; Idea of compartments in pharmacokinetics ; Compartmental Analysis technique ; Two-compartment model ---- clinical Bromsulphalein test ; Basic equations for an n-compartment system ; Distributions of drug in n-compartment model for (a) given initial injection, (b) repeated medication, (c) constant rate of infusion and (d) truncated infusion ; Repeated Penicillin application in a two-compartment model ; Compartment model for diabetes mellitus ; Stochastic compartment models ; Some general principles for real biological oscillations; Bifurcation theory and chaos.

Books Recommended:

E. C. Pielou : " An Introduction to Mathematical Ecology ", Wiley, New York.
R. W. Poole : " An Introduction to Quantitative Ecology ", McGraw-Hill.
R. Habermann : " Mathematical Models ", Prentice Hall.
J. N. Kapur : " Mathematical Models in Biology and Medicine ", East-West Press Pvt Ltd.
Robert C Hilborn: " Chaos and Nonlinear Dynamics: An introduction for Scientists and engineers," Oxford University Press (2009)
D.W.Jordan and P. Smith : "Nonlinear Ordinary Differential Equations: : An introduction for Scientists and engineers", Oxford University Press (2009)
J D Murray : "Mathematical Biology-Vol.-I", Springer (2002)
J D Murray : "Mathematical Biology-Vol.-II", Springer (2002)
Xiao-Qiang Zhao : "Dynamical Systems in Population Biology", Springer (2004)
Roy M. Anderson and Robert M May: " Infectious Diseases of Humans", Oxford University Press (2004)

MPS -3: MATHEMATICAL BIOLOGY-II

Mechanics of blood vessels:

Importance of studying the mechanics of blood vessels ; Structure and functions of blood vessels ; Mechanical properties of blood vessels ; Viscoelasticity ; Linear discrete viscoelastic (spring-dashpot) models : Maxwell Fluid, Kelvin solid, Kelvin chains and Maxwell models ; Creep Compliance and Relaxation Modulus ; Hereditary Integrals ; Stieltjes Integrals. Large deformation theory ; Strain energy function and its various forms ; Base vectors and metric tensors ; Green's deformation and Lagrangian strain tensors ; Cylindrical arterial model ; Constitutive equations for blood vessels ; Equations of motion for the blood vessel.

Mechanics of bio-fluid flow:

Importance of studying the mechanics of arterial flow ; Blood and its major constituents --- Structure and Functions ; Mechanical properties of blood ; Circulation of blood ; Equations of motion applicable to blood flow ; Non-Newtonian fluids --- power law, Bingham plastic, Harschel-Bulkley and Casson fluids ; Steady non-Newtonian fluid flow in a rigid circular tube under a constant pressure gradient ; Fahraeus-Lindqvist effect ; Models of stenosis and its effect ; Blood flow trough arteries with mild stenosis ; Pulsatile blood flow in both rigid and elastic arterial tubes ; Peristaltic flows in biomechanics ; Long wave length approximation to peristaltic flow in a tube ; Two dimensional flow in renal tubule : (a) Basic equations and boundary conditions , (b) Solution when radial velocity at wall decreases linearly with z , (c) Solution when radial velocity at wall decreases exponentially with z.

Diffusion and diffusion reaction models:

Importance of diffusion process in physiological system ; Fick's laws of diffusion ; Solutions of one-dimensional diffusion equation ; Solutions of two-dimensional diffusion equation ; Modifications of the diffusion equation ; Diffusion in artificial kidney (Hemodialyser): Function of Hemodialyser, Basic equations ; Solutions by (a) Separation of variables method, (b) Galerkin's approximation method ; Solution of the dialyser problem for non-Newtonian fluids ; The conjugate boundary value problem ; Oxygen diffusion through living tissues : the Krogh cylindrical model, Partial differential equations and boundary conditions ---(a) Capillary region, (b) Tissue region ; Krogh's steady state solution for tissue region ; Equation for motion of blood in capillary region ; Blum's steady state solution, Solution by Galerkin's method ; Comparison of oxygen diffusion problem with dialyser problem.

Books Recommended:

Y. C. Fung : " Biomechanics of Soft Biological Tissues ", Springer Verlag.
D. A. MacDonald : " Blood Flow in Arteries ", The Williams and Wilkins Company, Baltimore.
H. M. Lieberstein : " Mathematical Physiology ", American Elsevier Publishing Company, Inc.
J. N. Kapur : " Mathematical Models in Biology and Medicine ", East-West Press Pvt Ltd.
T J Pedley: " The Fluid Mechanics of Large Blood Vessels", Cambridge University Press (2008)

MPS-4: Functions of Complex Variables

Nevanlinna's Theory of Meromorphic Functions : Poisson-Jensen formula, Characteristic function, First fundamental theorem, Second fundamental theorem, Generalisations of the second fundamental theorem, Meromorphic functions with two Picard exceptional values.
Fix-points of Meromorphic Functions : Definition of fix-points, Some theorems of Rosenbloom on fix-points, Some theorems of Baker on fix-points.
Factorisation of Meromorphic Functions : Basic concepts and Definitions, Factorization of certain functions.
Fix-points and Theory of Factorisation : The relationship between the fix-points and theory of factorisation.
Unicity of Functions of Finite (lower) Order : Hadamard's factorisation theorem, Unicity of Meromorphic functions of order < 1, Functions of finite non-integer (lower) order.
Five-value, Multiple Value and Uniqueness : Nevanlinna's five-value theorem, Multiple values and Uniqueness.
The Four-Value Theorem : Nevanlinna's four-value theorem, 3CM+1IM value theorem, 2CM+2IM values theorem.

Functions of Several Complex Variables : Poly-cylindrical domain, Cauchy's integral formula, C-R criterion for analyticity, Taylor's theorem, Zeros and Singularities.

References:

Uniqueness Theory of Meromorphic Functions, Chung-Chun Yang and Hong-Xun Yi, Kluwer Academic Publishers, 2003.
Meromorphic Functions, W.K.Hayman, Oxford University press, 1964.
Fix-points and Factorisation of meromorphic Functions, Chi-Tai Chung and Chung-Chun Yang, World Scientific.
Elementary Theory of Analytic Functions of One or Several Complex Variables, H.Cartan, Dover Publications,INC, New York.

MPS-5: Fuzzy Functional Analysis

Fuzzy Arithmetic: Fuzzy real numbers, Representation of a fuzzy real number in tenns of a family of nested intervals. Arithmetic operations on fuzzy real numbers, Lattice of fuzzy real numbers, Fuzzy equations. (20)

Fuzzy Topology: Different types of fuzzy topology; Gradation of openness and closedness; Closure, interior; Closure operator. Base and sub-base of fuzzy topology; Continuity of mappings; Gradation preserving maps, homeomorphism. (30)

Fuzzy Metric Spaces: Different types of fuzzy metric (Kramosil & Michalek, like Deng, Kaleva & Seikkala). Underlying topology of a fuzzy metric space. Sequences-Cauchyness and convergence. Completeness of fuzzy metric spaces. Fixed point theorems (Banach, Edelstine etc.) in fuzzy metric spaces. (30)

Fuzzy Normed Linear Spaces: Different types of fuzzy nonns (Katsaras, Felbin, Cheng & Mordeson, Bag & Samanta).

Decomposition theorems; Convergence andCauchyness of sequences, Completeness of fuzzy nonned linear spaces, Equivalent fuzzy Donns.
Boundedness and continuity of linear operators over fuzzy nonned linear spaces. Fuzzy nonn of bounded linear operators, fuzzy dual space.
Some geometrical properties in fuzzy nonned linear spaces (viz. fuzzy radius, fuzzy Chebyshev radius, fuzzy Chebyshev centre, fuzzy nonnal structure etc.). Uniformly convex fuzzy nonned linear space. Some fixed point theorems in fuzzy nonned linear spaces (viz. Browder-Kirk, Schauder etc.). (40)

Books Recommended:

1. A. Kaufman, Introduction to the theory of fuzzy subsets (Academic Press).
2. G.J.Klir, B. Byan, Fuzzy Sets and Fuzzy Logic, Theory and Applications (Prentice Hall of India).
3. L.Y.Ming, L.M.Kang, Fuzzy Topology (World Scientific).
4. S.C.Cheng, J.N.Mordeson, Y. Yandong, Lecture Notes in Fuzzy Mathematics and Computer Science, Centre for Research in Fuzzy Mathematics, Creigton University, U.S.A.

MPS-6: Algebraic Theory of Semigroups

Congruences on a semigroup: (ker, tr) characterizations of congruences on inverse semigroups. Equivalence relations induced by ker and tr on the lattice of congruences on an inverse semigroup; idempotent separating congruences and idempotent pure congruences on an inverse semigroup. Group congruences on an inverse semigroup. The smallest inverse semigroup congruence on an orthodox semigroup.

Clifford decomposition of semigroups: Union of semigroups, band of semigroups, semilattice of semigroups. Characterization of semigroups which are union of groups; semilattice decompositions of completely regular semigroups into completely simple semigroups. Characterization of semigroups which are band of groups. Characterization of semigroups which are semilattice of groups as a strong semilattice of groups. A semigroup S is a Clifford semigroup if and only if it is a completely regular inverse semigroup.

Completely simple semigroups: Characterizations and examples. Representation of completely simple semigroups as a Rees matrix semigroup. Homomorphisms and congruences on completely simple semigroups.

Completely regular semigroups: A semigroup S is completely regular if and only if $a \in a^2Sa$ and other similar necessary conditions. A semigroup S is completely regular if and only if it is a union of groups. If a^{-1} is the inverse of a in the subgroup H_a and a^0 is the identity in H_a then for any congruence ρ on S, $a \rho b$ implies that $a^{-1} \rho b^{-1}$ and $a^0 \rho b^0$. Green's relations on a completely regular semigroup. Every regular subsemigroup of a completely regular semigroup is completely regular.

Bands; every band is a semilattice of rectangular bands. Every rectangular band is a direct product of a left zero band and a right zero band. Regular bands; the variety of regular bands has 13 subvarieties (statement only) . Elementary properties and semilattice decompositions of these particular regular bands.

Semilattice of Archimedean semigroups: Divisibility (left, right) in semigroups, radical of a subset in a semigroup. Semilattice of Archimedean semigroups, a semigroup is a semilattice of Archimedean semigroups if and only if it is a Putcha semigroup. A semigroup is Pucha if and only if radical of every ideal is an ideal. A semigroup *S* is a
chain of Archimedean semigroups if and only if it is semiprimary. Generalization of these semigroups into semilattice (chain) of left Archimedean semigroups and t-Archimedean semigroups.

Ordered semigroups: Ordered regular and intra-regular semigroups. Green's relations on an ordered semigroup. Ordered semigroups which are semilattice (chain) of (left, t-) simple semigroups.

References:

1. Clifford and Preston : The Algebraic Theory of Semiroups (Vol. I, II), Amer. Math. Society.
2. J. M. Howie: Fundamentals of semigroup Theory, Clarendon Press, Oxford.

3. M. Petrich : Introduction to semigroups, Merrill, Columbus.
4. M. Petrich : Inverse Semigroups, John Wiley & Sons.
5. M. Petrich : Lectures in Semigroups, John Wiley & Sons.
6. Petrich and Reilly : Completely regular Semigroups, John Wiley & Sons.

MPS-7: Fuzzy Algebra

Fuzzy Groups: Fuzzy subgroups, a subset H is a subgroup if and only if C_H is a fuzzy subgroup, necessary and sufficient condition for a fuzzy subset to be a fuzzy subgroup, both homomorphic image and inverse image of a fuzzy subgroup is a fuzzy subgroup, fuzzy subgroup generated by a fuzzy subset . Normal fuzzy subgroups, a subset N is a normal subgroup if and only if C_N is a normal fuzzy subgroup, necessary and sufficient condition for a fuzzy subset to be a normal fuzzy subgroup, normalizer of a fuzzy subset. Cosets of a fuzzy subgroup and the quotient group relative to the normal fuzzy subgroup, quotient fuzzy subgroup. Weak homomorphisms and weak isomorphisms, homomorphisms and isomorphisms, fuzzy isomorphism theorem. Complete and weak direct products. Fuzzy order relative to fuzzy subgroups, Fuzzy orders in cyclic groups. Fuzzy Cayley's Theorem and Fuzzy Lagrange's Theorem.

Fuzzy Rings: Fuzzy subrings and fuzzy ideals, a subset S is a subring (ideal) if and only if C_S is a fuzzy subring (ideal), necessary and sufficient condition for a fuzzy subset to be a fuzzy subring (ideal), fuzzy subring (ideal) generated by a fuzzy subset. Fuzzy cosets and the quotient ring relative to a fuzzy ideal. Isomorphism theorems. Fuzzy prime ideals, fuzzy maximal ideals and fuzzy semiprime ideals. Characterization of regularity.

Fuzzy Semigroups: Fuzzy (left, right) ideals in semigroups. Fuzzy bi-ideals, fuzzy interior ideals, fuzzy quasi-ideals, fuzzy generalized bi-ideals in semigroups. Rosenberg connection between ideals and fuzzy ideals through characteristic function. Fuzzy (left, right) simple semigroups; a semigroup S is simple if and only if it is fuzzy simple. A fuzzy subsemigroup f of a semigroup is a fuzzy bi-ideal if and only if $foSof$ is a subset of S. Intersection of a fuzzy left ideal and a fuzzy right ideal is a fuzzy quasi-ideal. A semigroup is a group if and only if every (bi-) quasi-ideal is a constant function. Fuzzy ideals generated by fuzzy subsets of a semigroup. Regular semigroups; Completely regular semigroups; Intra-regular semigroups; semisimple semigroups; Fuzzy regular subsemigroups of a semigroup. Characterizations of these semigroups by their fuzzy (bi-, quasi-, interior, left, right) ideals.

References:

1. Mordeson, Bhutani and Rosenfeld: Fuzzy Group Theory, Springer.
2. Rajesh Kumar: Fuzzy Algebra, University of Delhi.
3. Malik, Mordeson, Kuroki: Fuzzy Semigroups, Springer.

MPS-8: Fluid Mechanics

Dimensional analysis: Technique of dimensional analysis, Principle of dynamic similarity. Some useful dimensionless numbers such as Reynold's number, Froude number, Euler number, Mach number, Prandtl number, Eckert number, Peclet number, Weber number.
The velocity potential and Laplace's equation. Simple irrotational flows. Solutions by separation of variables, Separation of variables for an ,axisymmetric flow: Legendre polynomials. Singularities of flow, Source, Sink, Doublets, Rectilinear vortices. Complex variable method for two-dimensional problems.
Exact solutions of the Navier-Stokes equations; Steady plane flows: Couette -Poiseuille flows, Jeffery-Hamel flows, Plane stagnation point flow. Steady &xisymmetric flows: Circular pipe flow (Hagen-Poiseuille flow), Flow between two concentric cylinders. Unsteady plane flows: First Stokes problem and Second Stokes problems. Oscillating channel flow. Unsteady pipe flow.
Boundary layer theory: Concept of boundary layer. Laminar boundary layer on a flat plate at zero incidence, Estimation of the boundary layer thickness, Displacement thickness. Separation of the boundary layer. Prandtl's boundary layer equation. Similiarity solutions of boundary layer equations. Blasius equation for steady 2-D flow past a flat plate.
Numerical Integration of the boundary layer equation: Laminar boundary layers, Boundary layer transformations, Explicit and implicit Discretisation. Integration of the Transformed boundary layer equations using Box scheme.
Three dimensional flow, Irrotational motion, Weiss's theorem and its applications. Viscous flow, Vorticity dynamics, Vorticity equation, Reynolds number, Stress and strain analysis, Navier-Stokes equation.
Stability theory: Fundamental of stability theory, Orr-Sommerfeld equation. Transition in the pipe flow, Transition in the boundary layer.

Reference Books:
Frank M White, Viscous Fluid Flow, McGraw-Hill, 1991.
Schlichting and Gersten, Boundary-Layer Theory. Springer-Verlag, 2000.
G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 1993. . F. Chorlton, A Text Book of Fluid Dynamics, Von Nostrand Reinhold/CBS, 1985.
R. Patterson, A First Course in Fluid Dynamics, Cambridge University Press, 1992.
H. Lamb, Hydrodynamics, Cambridge University Press.

MPS- 9: Advanced Real Analysis

Measure theory and integration on general spaces. Egoroff's theorem. Lusin's theorem. Convergence theorems. Product measures and Fubini's theorem. Regular Borel measures on compact spaces, Riesz - Markov theorem (Riesz representation theorem for the space of continuous functions).
Signed measures, Hahn-decomposition theorem. Jordan decomposition theorem. Radon-Nidodym theorem. Radon-Nikodym derivative. Lebesgue decomposition theorem. Complex measure. Integrability of functions w.r .t. signed measure and complex measure.
Measurable Rectangles, Elementary sets. Product measures. Fubini's theorem.
Lp [a,b] - spaces ($1 \leq p \leq \alpha$). Holder and Minkowski inequality. Completeness and other

properties of Lp[a, b] spaces. Dense subspaces of Lp[a, b] - spaces. Bounded linear functionals on Lp[a, b] - spaces and their representations.
Density of arbitrary linear sets. Lebsegue density theorem. Approximate continuity. Properties of approximately continuous functions. Bounded approximately continuous function over [a,b] and exact derivative.
The Perron integral: Definitions and basic properties, Comparison with Lebesgue integral and Newton integral.
Trigonomeric system and Trigonometric Fourier series. Summability of Fourier series by (C, I), means. Fejer's theorem. Lebesgue's theorem. Completeness of Trigonometric system.
Sets of the 1st and of the 2nd categories. Baires theorem for G_δ residual and perfect sets.
Baire classification of functions. Functions of Baire class one. Baire's theorem. Semicontinuous functions.

MPS-10: Plasma Dynamics

Basic concepts: Maxwell's Equation. Definition of Plasma. Concept of Debye length, gyro radius, cyclotron frequency, collision parameters in plasma.
Motion of a charged particle in electric and magnetic field. Drift velocity.
Waves and oscillations, dispersion relation, instabilities (Two stream instability and Jeans instability) in plasma.
Kinetic theory: Boltztmann equation, Vlasov equation and Liouville equation. Transport phenomena. Landau Damping.
Burger's equation, KdV equation, KP equation, ZK equation, NLS equation.
Reductive perturbative technique and derivation of the above equations in different plasma environment. Sagdeev's pseudo-potential technique, KBM method, Inverse scattering technique.
Introduction to dusty plasma and quantum plasma.
Solitons, shocks,vortices and other nonlinear structures in plasma.

References:
1. Introduction to Plasma Physics - F F Chen, (New York, Plenum)
2. Introduction to un-magnetized Plasma - Chanchal Uberoi.(prentice hall)
3. Introduction to Dusty Plasma Physics - P. K. Shukla and A. A. Mamun. (lOP, Bristol)
4. Physics of Solitons - Cambridge University Press.
5. Nonlinear waves, solitos and chaos, Eryk Infeld and George Rowlands- Cambridge University Press.
6. R. j. Goldstein, P. H. Rutherford, Plasma Physics (lop, Bristol, 1995). 7. S. Ichimaru, Plasma Physics (Benjamin, Menlo park).

MPS-11: Meteorology of Air Pollution

Basic concepts: Air pollution system, Air pollutants, Surface layer, Mixing layer, Air quality models, Effects of air pollution.
Air pollution chemistry: Atmospheric energy balance, Atmospheric photochemical reactions; Reactions of Nitrogen oxides, hydrocarbons and sulphur oxides in the urban atmosphere; Photochemical smog, Aerosol process in the urban atmosphere.
Micrometeorology: Basic equations of atmospheric fluid mechanics, Equations for the mean

quantities, Mixing-length models for turbulent transport, Variations of wind with height in the atmosphere, Meteorological measurements important in air pollution, dry and wet depositions, Atmospheric stability.
Atmospheric Diffusion: General description of turbulent diffusion; Equations governing the mean concentration of species in turbulence, Solutions of the atmospheric diffusion equations, Stability theory for diffusion in the atmospheric surface layer, Diffusive transport in the surface layer.
Plume rise: Buoyant plumes in a calm atmosphere, Forced plumes in a calm atmosphere; Plume rise in a windy atmosphere, Plume rise formulas compared with observations.
Acid Rain: Cloud physics, solubility of gases, Transfer of gases to liquids, Composition of precipitation, Chemistry inside droplets, Acid deposition.

Reference Books

1. M.N. Rao et al., Air Pollution, Tata Mc.Graw Hill, 1998.
2. Noel de Nevers, Air Pollution Control Engineering, Mc.Graw Hill, New York. 1995.
3. A.C. Stern, Air Polluti,on, Vol.I, II and III, Academic Press, 1962.
4. J.H.. Seinfeld, Atmospheric Physics and Chemistry of Air Pollution, Wiley, New York (1985).

MPS-12: Computational Fluid Dynamics

Classification of second order PDE's. Dirichlet, Neumann and Robin's problem.
Derivations of finite difference equations, Multidimensional finite difference formulas, Mixed derivatives, Accuracy of finite difference solutions, Higher order accurate schemes.
Basic finite difference schemes for elliptic, parabolic and hyperbolic equations. Upwind scheme, Consistency, CFL stability condition, Convergence and stability analysis, Explicit and implicit methods. Convection-diffusion equation. ADI, Lax-Wendrofand MaCormack schemes. Thomas algorithm.
Different forms of Navier-Stokes equations. Fundamental equations in cylindrical and spherical coordinates. Biharmonic formulation of incompressible viscous flows. Incompreessible viscous flows via finite difference methods, Artificial compressibility method, Pressure correction method: SIMPLE and SIMPLER algorithm. Vortex method. .
Grids with appropriate transformation: General transformation of the equations, Metrics and Jacobians, Form of the governing equations particularly Suite for CFD revisited: The transformed version, Stretched grids, Boundary fitted coordinate systems; Elliptic grid generation.
Adaptive methods: Structured Adaptive methods, Unstructured adaptive method.
Modes of heat transfer; Conduction: I-D, 2-D, and 3-D steady conduction, l-d unsteady conduction - analytical/numerical solution methods. Convection: fundamentals, order of magnitude analysis of ' momentum and energy equations, free and forced convection.
Application of numerical methods for solving conjugate radiation, conduction and/or convection problems in I-D and 2-D Cartesian and axis-symmetric geometry.

Reference Books:

D. A. Anderson, J. C. Tannehill and R. H. Pletcher, Computational Fluid Dynamics and Heat Transfer, McGraw Hill, 1984.
Joe F. Thompson, Z. U. A. Warsi, C. Wayne Mastin, Numerical grid generation.

F.P. lncropera and D.P. Dewitt, Fundamentals of Heat and Mass Transfer, 4e, John Wiley and Sons. 1996.
R. Siegel and J. R. Howell, Thermal Radiation Heat Transfer, 3rd edition, Taylor and Francis, 1992.
P. Niyogi, Introduction to Computational Fluid Dynamics, Pearson Publishers.
Joe D. Hoffman, Numerical methods for Engineers and Scientists, McGrow Hill.
J. D. Anderson, Jr, Computational fluid dynamics-the basics with applications. McGrow ~(ill.

MPS-13: Introduction to Topological Dynamics

Topological groups and semigroups: Elementary concepts in topological groups and semigroups.
Basic concepts and examples: Discrete and continuous time dynamical systems. Flow, semiflow, transformation groups. Basic concepts and examples. Topological semiconjugacy and conjugacy.
Maps: Continuous maps on a metric space; Iterates, orbits, fixed points, periodic points and their stability. Attractors and repellors. Graphical analysis of orbits of one dimensional maps. Period two points and bifurcations. Introduction to topological transitivity and sensitive dependence on initial conditions. Topological entropy; topological entropy is topological conjugacy invariant.
Transformation groups: Transformation group (X, G), phase space and the phase group. Invariant subsets and subsystems, minimal sets. Orbit, orbit closure, transitive points and minimal or almost periodic points. Syndetic subsets of the phase group, Inheritance theorem.
Minimal, Proximal and distal systems: Ellis-Nakamura Theorem: for a compact Housdorff topological semigroup L, if a→ab is continuous then L has an idempotent. The Ellis semigroup or enveloping semigroup E(X) of a transformation group. Ex is the orbit closure of x. M is a minimal subset if and only if M is a minimal left ideal of E. x is a minimal point if and only orbit closure of x is Mx for every minimal ideal M of E if and only if every minimal ideal M of E contains an idempotent e such that ex=x. E consists of continuous maps if and only if (X,G) is weakly almost periodic. Proximal and distal pair, proximal relation, proximal to a subset. In a compact Housdorff system every point is proximal to an almost periodic point in its orbit closure. Distal points and distal dynamical system, every distal point is almost periodic. Every distal system is a union of minimal sets.
Topological Transitivity, topological mixing: Equivalence of these two, Equivalent conditions of topological transitivity and of topological mixing.
Sensitive dependence on initial conditions and chaos: Metrizability of a topological space, Glassner characterizations of the sensitive dependence on initial conditions. Characterizations of chaos by Devaney.

References:
Brin and Stuck: Introduction to Dynamical Systems, Cambridge.
J. de Vries: Elements of topological Dynamics, Kluwer.
R. Devaney: Chaotic Dynamical Systems, Addison Wesley.
R. Ellis: Lectures on topological Dynamics, W. A. Benjamin.
Paalman De Miranda: Topological Semigroups, Mathematisch Centrum Amsterdam.
P. J. Higgins: Topological groups, Oxford.

MPS-14: Uncertainty – Theory of Possibility, Plausibility and Usuality

Concept of probability measure—role of probability theory in handling uncertainty. Measure of possibility—natural examples. Possibility distributions---their representation and fuzzy set connection. Role of possibility measure in imprecise knowledge representation and manipulation. Translating rules----modification,composition,quantification and qualification. Consistency, compatibility and truth of sentences. PRUF (Possibilistic Relational Universal Fuzzy)---the meaning representation language. Examples of translation into PRUF.

Plausibility measures ---natural examples. Role of plausibility measure in uncertain knowledge representation and manipulation.

Usuality qualified propositions --- meaning representation. Its relation with default and typical values. Reasoning with usuality qualified propositions. Natural languages.

References:

1. D.Dubois, H.Prade, Possibility theory, Plenum Press, New York, 1988
2. R.Kruse, J.Gebhardt, F.Klawonn, Foundations of fuzzy systems, Wiley, New York, 1994.
3. S.Gottwald, Fuzzy sets and Fuzzy logic, Vieweg ,Wiesbaden, 1993.
4. L.A.Zadeh, PRUF-a meaning representation language for natural languages, International Journal of man-machine studies, vol. 10,1978, pp.395-460.
5. L.A.Zadeh. outline of a theory of usuality based on fuzzy logic, in A. Jones et.al. Fuzzy sets : theory and applications, pp.79-97.
6. J.C. Bezdek, Computing with uncertainty, IEEE communications magazine, vol. 30, 1992, pp.24-36.

MPS-15: Formal Language, Grammar and Automata

Grammar and Language; classification of grammars in terms of their production and the language they define --- Chomsky hierarchy.
Sequential machine without output --- definition, response function, accessibility of states, connected machine, congruence relation, quotient machine and homomorphism of machines. Sequential machine with output --- definition and basic properties. Behaviour of a sequential machine, behavioural equivalence. Finite automaton. Minimal sequential machine having a given behaviour, relation on the set of states, algorithm for computing minimal machine.
Transition system and regular expression --- the subset construction. Language of regular expression. Analysis and synthesis theorems and their applications. Hardware realization of sequential machine. Decomposition of machine.
Definite event, definite set, definite machine and definite transition function. Test for definiteness. Probabilistic machine, behaviour of a probabilistic machine, Rabin's motivation for the introduction of isolated cut-points, actual probabilistic machine.
Turing machine --- unrestricted grammar; linear bounded automaton --- context- sensitive grammar; Pushdown automaton --- context-free grammar and Regular automaton --- regular grammar.

References:
1. H.R.Lewis and C.H.Papadimitriou --- Elements of the theory of computation; Prentice-Hall.
2. J.E.Hopcroft and J.D.Ullman --- Introduction to automata theory, languages and computation; Addison- Wesley.
3. S.Ginsberg --- An introduction to mathematical machine theory; Addison- Wesley.
4. M.A.Harrison --- Introduction to switching and automata theory; McGraw-Hill.
5. A.Paz --- Introduction to probabilistic automata; Academic Press.

MPS-16: Quantum Scattering Theory

The quantum theory of scattering of a beam of particles by a centre of force – definitions of differential and total scattering cross sections, low-energy scattering by the method of partial waves, behavior of phase shifts for different potentials, Jost function, scattering matrix, reactance matrix, examples.

Scattering by a centre of force – Green's function, the Born approximation, convergence of Born approximations, classical limit of the quantum theory scattering formulas, semi-classical approximations, examples.

Variational methods for central force problems – Kohn and Hulthen variational methods for determining the phase shifts, bounds on scattering lengths and phase shifts, methods for calculating the scattering amplitude – Schwinger variational method, examples.

General theory of atomic collisions – collisions of electrons and positrons with hydrogen atoms, Green's function, expressions for the scattering amplitude and cross sections, the Born approximation and its validity, the impulse approximation and its validity, the two-state approximations, the distorted-wave models, examples of elastic collisions, excitation and ionization processes.

General theory of rearrangement collisions – capture of an atomic electron by an incident positron or a proton, two-state approximation - distorted-wave models for intermediate and high energy collisions, low-energy approximations – the Schwinger variational method, examples.

Relativistic scattering of a charged particle from a center of force in the presence of an external electromagnetic field. Analysis to describe the scattering of a Dirac particle. Solutions of the wave equation describing the asymptotic motion of the particle in the presence of the field.

References:
1. N F Mott and H S W Massey – Theory of Atomic Collisions (Oxford: Clarendon Press, 1965) 3rd Ed.
2. M L Goldberger and K M Watson – Collision Theory (New York: John Wiley & Sons, Inc., 1964)
3. R G Newton – Scattering Theory of Waves and Particles (New York: McGraw-Hill, 1966)
4. C J Joachain – Quantum Collision Theory (New York: North-Holland, 1979)
5. B L Moiseiwitsch – Variational Principles (New York: John Wiley & Sons, Inc., 1966)